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SIMPLE AND EFFICIENT NUMERICAL TECHNIQUES FOR TREATING BODIES OF REVOLUTION

University of Mississippi

A. W. Glisson
D. R. Wilton

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ROME AIR DEVELOPMENT CENTER
Air Force Systems Command
Griffiss Air Force Base, New York 13441

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APPROVED: Jacob Scherer
JACOB SCHERER
Project Engineer

APPROVED: Joseph J. Naresky
JOSEPH J. NARESKY
Chief, Reliability and Compatibility Division

FOR THE COMMANDER: John P. Huss
JOHN P. HUSS
Acting Chief, Plans Office

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Integral equations are derived and a numerical procedure is developed to treat scattering by bodies of revolution. Both dielectric and metallic scatterers are considered and the metallic scatterers may be either open or closed. The program is validated by comparing to spheres and finite cylinders for the dielectric case. In the conducting case, a cone-spheres and an open-ended cylinder are treated. It is found that the problem of instability noted in other formulations for the open-ended cylinder problem does not appear in the present formulation. A brief description of the program and a program —

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INTRODUCTION

The determination of the skin currents induced on a missile in flight is an important first step in predicting the response of internal subsystems to an impressed electromagnetic field. The missile skin currents, however, may be affected by the presence of the rocket exhaust (plume) which consists of partially ionized gases or plasma and therefore has a finite conductivity. Because of the difficulty and expense involved in obtaining measured data, a reasonably accurate numerical solution procedure is desirable. Furthermore, numerical results can provide a partial validation of actual experiments. The more sophisticated numerical analyses generally involve a body of revolution model for both the missile and the plume. In this report we consider body of revolution models in general, for both perfect electric conductors and dielectric bodies, without reference to a particular structure.

The primary impetus for the development of the computer code described in this work has been the desire for simplicity of formulation and for increased accuracy and efficiency. Other numerical formulations for bodies of revolution are

already available [1, 2, 3] and, indeed, the formulation of [3] has been generalized to the missile/plume problem and results have been presented in a recent report (Wu, et. al. [3]). However, some stability problems have been encountered in these formulations for certain geometries, whereas the formulation presented in Section II has been extensively tested and has demonstrated no apparent stability problems. More importantly, however, this formulation and computer code serves as a basic building block from which an extremely complicated code is to be developed which treats a very sophisticated model of the missile/plume structure in which the inhomogeneous plume is modeled by layers of homogeneous material. This formulation will be described in a forthcoming report.

The numerical solution procedure for a body of revolution, which may be either a perfectly conducting body or a dielectric body, is presented in Section II. Numerical results for several cases are also presented and discussed. In Appendix B the implementation of the numerical formulation is discussed with direct reference to an available computer code which is also listed in the appendix.

Section II

NUMERICAL SOLUTION PROCEDURE FOR A BODY OF REVOLUTION

In this section integral equations are derived for equivalent surface currents induced on a dielectric body of revolution subject to plane wave illumination. Scattering by a perfectly conducting body of revolution can also be obtained as a special case. The numerical solution procedures are described and the method of moments is applied. Numerical results are obtained for several cases and are compared with other available data.

2.1 Formulation of the Integral Equations

Consider the body of revolution of Fig. 2.1, where the body is formed by rotating a planar curve, called the "generating arc," around the z-axis (also called the axis of the body of revolution). The regions exterior and interior to the body are denoted as regions 1 and 2, respectively. The t-coordinate, which is depicted in Fig.2.1, follows the generating arc on the body surface S. The body, with constitutive parameters (μ_2, ε_2, $\sigma_2 = 0$), is considered

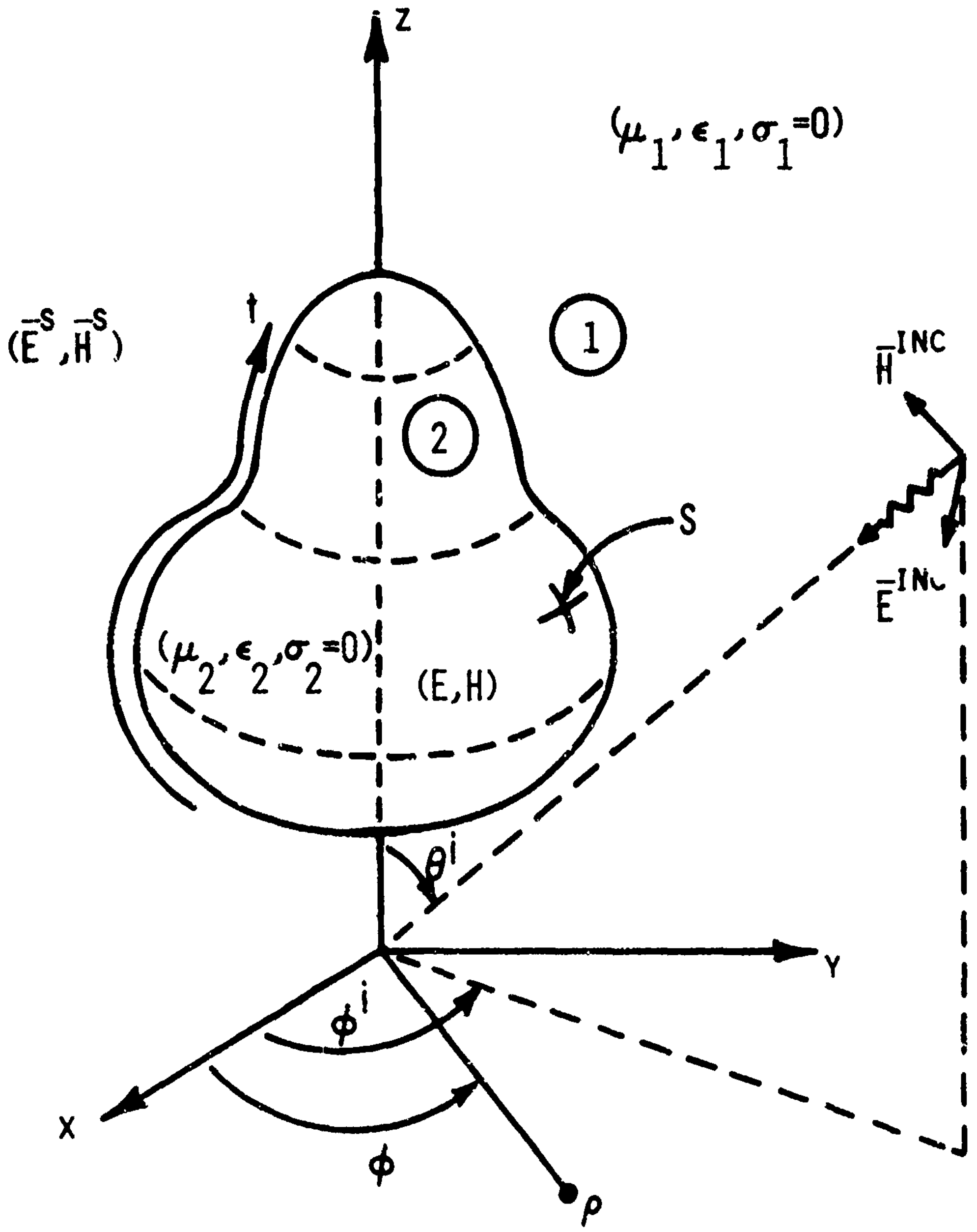

Figure 2.1. Geometry of the body of revolution.

to be immersed in an infinite homogeneous medium with parameters (μ_1, ε_1, $\sigma_1 \to 0$). The generalization to non-zero conductivity of either the body or exterior medium parameters is trivial, but has not been considered here.

Boundary conditions require that the total fields tangential to the surface of the body be continuous from Region 1 to Region 2. These conditions may be expressed as

$$\hat{n} \times (\overline{E} - \overline{E}^s) = \hat{n} \times \overline{E}^{inc} \tag{2.1a}$$

$$\hat{n} \times (\overline{H} - \overline{H}^s) = \hat{n} \times \overline{H}^{inc} \quad , \tag{2.1b}$$

where ($\overline{E}^{inc}$, $\overline{H}^{inc}$) constitute the incident fields, ($\overline{E}^s$, $\overline{H}^s$) are the scattered fields in Region 1, and ($\overline{E}$, $\overline{H}$) are the total fields in region 2, and $\hat{n} = \hat{\phi} \times \hat{t}$ is the outward unit normal to the body surface. Via the equivalence principle, the body is replaced by two sets of electric currents ($\overline{J}_i$, $i = 1,2$) and magnetic currents ($\overline{M}_i$, $i = 1,2$) — one set just inside the surface, and the other just outside the surface. Each of the two sets of currents radiates in an infinite homogeneous medium having the constitutive parameters with the corresponding medium index, $i = 1,2$. Thus the fields indicated in (2.1) may be expressed as

$$\overline{E}^s(\overline{r}) = -j\omega\overline{A}_1(\overline{r}) - \nabla\Phi_1(\overline{r}) - \frac{1}{\varepsilon_1} \nabla \times \overline{F}_1(\overline{r}) \tag{2.2a}$$

$$\overline{H}^s(\overline{r}) = -j\omega\overline{F}_1(\overline{r}) - \nabla\Psi_1(\overline{r}) + \frac{1}{\mu_1}\nabla \times \overline{A}_1(\overline{r}) \tag{2.2b}$$

$$\overline{E}(\overline{r}) = -j\omega\overline{A}_2(\overline{r}) - \nabla\Phi_2(\overline{r}) - \frac{1}{\varepsilon_2}\nabla \times \overline{F}_2(\overline{r}) \tag{2.2c}$$

$$\overline{H}(\overline{r}) = -j\omega\overline{F}_2(\overline{r}) - \nabla\Psi_2(\overline{r}) + \frac{1}{\mu_2}\nabla \times \overline{A}_2(\overline{r}) \ , \tag{2.2d}$$

where the potentials are defined by

$$\overline{A}_i(\overline{r}) = \frac{\mu_i}{4\pi}\iint_S \overline{J}_i(\overline{r}')G_i(\overline{r}, \overline{r}')\ dS' \tag{2.3a}$$

$$\overline{F}_i(\overline{r}) = \frac{\varepsilon_i}{4\pi}\iint_S \overline{M}_i(\overline{r}')G_i(\overline{r}, \overline{r}')\ dS' \tag{2.3b}$$

$$\Phi_i(\overline{r}) = \frac{1}{4\pi\varepsilon_i}\iint_S \rho_i^e(\overline{r}')G_i(\overline{r}, \overline{r}')\ dS' \tag{2.3c}$$

$$\Psi_i(\overline{r}) = \frac{1}{4\pi\mu_i}\iint_S \rho_i^m(\overline{r}')G_i(\overline{r}, \overline{r}')\ dS' \ , \qquad i = 1,2 \ , \tag{2.3d}$$

and where

$$G_i(\bar{r}, \bar{r}') = \frac{e^{-jk_iR}}{R} \quad , \qquad (2.4a)$$

$$i = 1,2 ,$$

$$R = |\bar{r} - \bar{r}'| = \left[\rho^2 + \rho'^2 - 2\rho\rho'\cos(\phi - \phi') + (z - z')^2\right]^{\frac{1}{2}} \quad . \qquad (2.4b)$$

The continuity of the tangential fields at the boundary requires that

$$\bar{J}_1 = -\bar{J}_2 \equiv \bar{J} \quad . \qquad (2.5a)$$

$$\bar{M}_1 = -\bar{M}_2 \equiv \bar{M} \quad . \qquad (2.5b)$$

Note that we have introduced unsubscripted currents $\bar{J}$ and $\bar{M}$. One may then also define an unsubscripted charge by

$$\rho^e \equiv \rho_1^e = -\rho_2^e = \frac{j}{\omega}[\nabla_s' \cdot \bar{J}(\bar{r}')] \qquad (2.6a)$$

$$\rho^m \equiv \rho_1^m = -\rho_2^m = \frac{j}{\omega}[\nabla_s' \cdot \bar{M}(\bar{r}')] \quad . \qquad (2.6b)$$

Combining Eqs. (2.1) through (2.6) allows one to write two vector integro-differential equations which may be enforced on the surface of the body to obtain the unknown electric and magnetic currents:

$$\hat{n} \times \overline{E}^{inc} = \hat{n} \times \left\{ \frac{j\omega}{4\pi} \iint_S \overline{J} \, (\mu_1 G_1 + \mu_2 G_2) \, dS' \right.$$

$$+ \frac{j}{4\pi\omega} \nabla \iint_S (\nabla_s' \cdot \overline{J}) \left[\frac{G_1}{\varepsilon_1} + \frac{G_2}{\varepsilon_2} \right] dS'$$

$$\left. + \frac{1}{4\pi} \nabla \times \iint_S \overline{M} \, (G_1 + G_2) \, dS' \right\} \tag{2.7a}$$

$$\hat{n} \times \overline{H}^{inc} = \hat{n} \times \left\{ \frac{j\omega}{4\pi} \iint_S \overline{M} \, (\varepsilon_1 G_1 + \varepsilon_2 G_2) \, dS' \right.$$

$$+ \frac{j}{4\pi\omega} \nabla \iint_S (\nabla_s' \cdot \overline{M}) \left[\frac{G_1}{\mu_1} + \frac{G_2}{\mu_2} \right] dS'$$

$$\left. - \frac{1}{4\pi} \nabla \times \iint_S \overline{J} \, (G_1 + G_2) \, dS' \right\} , \tag{2.7b}$$

where the dependence of the appropriate quantities on source and/or field coordinates is understood. Since the term involving the curl operator appearing in (2.7) is not continuous at the boundary, the direct interchange of integration and differentiation is not allowed. It can be shown [4],

however, that on the surface S,

$$\nabla \times \iint_S \overline{U}(G_1 + G_2)\ dS' = -⨏⨏_S \overline{U} \times \nabla(G_1 + G_2)\ dS' \quad , \qquad (2.8)$$

$$\overline{U} = \overline{J} \text{ or } \overline{M} ,$$

where $⨏⨏_S$ represents a Cauchy Principal Value integral. Eqs. (2.7) can be written in a component operator form as

$$E_t^{inc} = \beta_{11}(J_t) + \beta_{12}(J_\phi) + \beta_{13}(M_t) + \beta_{14}(M_\phi) \qquad (2.9a)$$

$$E_\phi^{inc} = \beta_{21}(J_t) + \beta_{22}(J_\phi) + \beta_{23}(M_t) + \beta_{24}(M_\phi) \qquad (2.9b)$$

$$H_t^{inc} = \beta_{31}(J_t) + \beta_{32}(J_\phi) + \beta_{33}(M_t) + \beta_{34}(M_\phi) \qquad (2.9c)$$

$$H_\phi^{inc} = \beta_{41}(J_t) + \beta_{42}(J_\phi) + \beta_{43}(M_t) + \beta_{44}(M_\phi) \quad , \qquad (2.9d)$$

where β_{ij} is the appropriate integro-differential operator. It is desirable to express all of the quantities of (2.9) in terms of the local coordinates (t,ϕ) on the body surface. An orthogonal triad of unit vectors $(\hat{n},\hat{\phi},\hat{t})$ may also be associated with each coordinate point (t,ϕ), where $\hat{n}$, $\hat{\phi}$, and $\hat{t}$ are defined as follows:

$$\hat{n} = \cos\gamma\cos\phi\,\hat{x} + \cos\gamma\sin\phi\,\hat{y} - \sin\gamma\,\hat{z} \qquad (2.10a)$$

$$\hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y} \qquad (2.10b)$$

$$\hat{t} = \sin\gamma\cos\phi\,\hat{x} + \sin\gamma\sin\phi\,\hat{y} + \cos\gamma\,\hat{z} \quad , \qquad (2.10c)$$

where γ is the angle between the tangent to the generating curve, $\hat{t}$, and the z-axis, defined to be positive if $\hat{t}$ points away from the z-axis and negative if $\hat{t}$ points toward the z-axis. We note that the surface divergence in this coordinate system becomes

$$\nabla_s' \cdot \overline{U} = \frac{1}{\rho'} \frac{\partial}{\partial t'}(\rho' U_t) + \frac{1}{\rho'} \frac{\partial}{\partial \phi'}(U_\phi) \quad , \qquad \overline{U} = \overline{J} \text{ or } \overline{M} . \tag{2.11}$$

One may now expand (2.7) into components and compare to (2.9) to obtain expressions for the β_{ij}:

$$\beta_{11}(J_t) = \frac{j\omega}{4\pi} \iint_S J_t [\sin\gamma \sin\gamma' \cos(\phi'-\phi) + \cos\gamma \cos\gamma'] \{\mu_1 G_1 + \mu_2 G_2\} \, dS'$$

$$+ \frac{j}{4\pi\omega} \frac{\partial}{\partial t} \iint_S \frac{1}{\rho'} \frac{\partial}{\partial t'}(\rho' J_t) \left\{ \frac{G_1}{\varepsilon_1} + \frac{G_2}{\varepsilon_2} \right\} dS' \tag{2.12a}$$

$$\beta_{12}(J_\phi) = -\frac{j\omega}{4\pi} \iint_S J_\phi \sin\gamma \sin(\phi'-\phi) \{\mu_1 G_1 + \mu_2 G_2\} \, dS'$$

$$+ \frac{j}{4\pi\omega} \frac{\partial}{\partial t} \iint_S \frac{1}{\rho'} \frac{\partial}{\partial \phi'}(J_\phi) \left\{ \frac{G_1}{\varepsilon_1} + \frac{G_2}{\varepsilon_2} \right\} dS' \tag{2.12b}$$

$$\beta_{13}(M_t) = -\frac{1}{4\pi}\iint_S \frac{M_t}{R}[(\rho'\sin\gamma\cos\gamma' - \rho\cos\gamma\sin\gamma')\sin(\phi'-\phi)$$

$$+ (z-z')\sin\gamma\sin\gamma'\sin(\phi'-\phi)] \frac{d}{dR}\{G_1+G_2\} \; dS' \qquad (2.12c)$$

$$\beta_{14}(M_\phi) = -\frac{1}{4\pi}\iint_S \frac{M_\phi}{R}[\rho'\cos\gamma - \rho\cos\gamma\cos(\phi'-\phi)$$

$$+ (z-z')\sin\gamma\cos(\phi'-\phi)] \frac{d}{dR}\{G_1+G_2\} \; dS' \qquad (2.12d)$$

$$\beta_{21}(J_t) = \frac{j\omega}{4\pi}\iint_S J_t\sin\gamma'\sin(\phi'-\phi)\{\mu_1 G_1+\mu_2 G_2\} \; dS'$$

$$+ \frac{j}{4\pi\omega\rho} \frac{\partial}{\partial\phi}\iint_S \frac{1}{\rho'} \frac{\partial}{\partial t'}(\rho' J_t) \left\{\frac{G_1}{\varepsilon_1} + \frac{G_2}{\varepsilon_2}\right\} \; dS' \qquad (2.12e)$$

$$\beta_{22}(J_\phi) = \frac{j\omega}{4\pi}\iint_S J_\phi\cos(\phi'-\phi)\{\mu_1 G_1+\mu_2 G_2\} \; dS'$$

$$+ \frac{j}{4\pi\omega\rho} \frac{\partial}{\partial\phi}\iint_S \frac{1}{\rho'} \frac{\partial}{\partial\phi'}(J_\phi) \left\{\frac{G_1}{\varepsilon_1} + \frac{G_2}{\varepsilon_2}\right\} \; dS' \qquad (2.12f)$$

$$\beta_{23}(M_t) = -\frac{1}{4\pi}\iint_S \frac{M_t}{R}[\rho\cos\gamma' - \rho'\cos\gamma'\cos(\phi'-\phi)$$

$$- (z-z')\sin\gamma'\cos(\phi'-\phi)]\ \frac{d}{dR}\{G_1+G_2\}\ dS' \qquad (2.12g)$$

$$\beta_{24}(M_\phi) = -\frac{1}{4\pi}\iint_S \frac{M_\phi}{R}(z-z')\sin(\phi'-\phi)\ \frac{d}{dR}\{G_1+G_2\}\ dS' \quad , \qquad (2.12h)$$

where R is defined by (2.4) and

$$\frac{dG_i}{dR} = -\frac{(1+jk_iR)}{R^2}\, e^{-jk_iR} \quad , \qquad (2.13)$$

$$i = 1,2 \ .$$

Eqs. (2.12) define the β_{ij} for (2.9a) and (2.9b). Recall that $\beta_{ij}(U)$ is an operator on the function U. One may also consider β_{ij}, $i = 1,2$, $j = 1,2$, to be dependent on the parameters of the media, i.e.

$$\beta_{ij}(U) = \beta_{ij}(U;\ \mu_1,\ \varepsilon_1,\ \mu_2,\ \varepsilon_2) \quad , \qquad i = 1,2\ , \quad j = 1,2\ .$$

Similarly,

$$\beta_{ij}(U) = \beta_{ij}(U;\ \mu_1,\ \varepsilon_1,\ \mu_2,\ \varepsilon_2) \quad , \qquad i = 3,4\ , \quad j = 3,4\ .$$

One then finds that

$$\beta_{33}(M_t;\ \mu_1,\ \varepsilon_1,\ \mu_2,\ \varepsilon_2) = \beta_{11}(M_t;\ \varepsilon_1,\ \mu_1,\ \varepsilon_2,\ \mu_2) \quad (2.14a)$$

$$\beta_{34}(M_\phi;\ \mu_1,\ \varepsilon_1,\ \mu_2,\ \varepsilon_2) = \beta_{12}(M_\phi;\ \varepsilon_1,\ \mu_1,\ \varepsilon_2,\ \mu_2) \quad (2.14b)$$

$$\beta_{43}(M_t;\ \mu_1,\ \varepsilon_1,\ \mu_2,\ \varepsilon_2) = \beta_{21}(M_t;\ \varepsilon_1,\ \mu_1,\ \varepsilon_2,\ \mu_2) \quad (2.14c)$$

$$\beta_{44}(M_\phi;\ \mu_1,\ \varepsilon_1,\ \mu_2,\ \varepsilon_2) = \beta_{22}(M_\phi;\ \varepsilon_1,\ \mu_1,\ \varepsilon_2,\ \mu_2) \quad (2.14d)$$

and

$$\beta_{31}(J_t) = -\beta_{13}(J_t) \quad (2.14e)$$

$$\beta_{32}(J_\phi) = -\beta_{14}(J_\phi) \quad (2.14f)$$

$$\beta_{41}(J_t) = -\beta_{23}(J_t) \quad (2.14g)$$

$$\beta_{42}(J_\phi) = -\beta_{24}(J_\phi) \quad . \quad (2.14h)$$

Thus Eqs. (2.14), which are actually statements of duality [5], serve to define the β_{ij} in (2.9c) and (2.9d) via Eqs. (2.12). Furthermore, if the body is a perfect conductor, (2.9) reduces to

$$E_t^{inc} = \beta_{11}(J_t;\mu_1,\varepsilon_1,\mu_2=0,\varepsilon_2=\infty) + \beta_{12}(J_\phi;\mu_1,\varepsilon_1,\mu_2=0,\varepsilon_2=\infty) \quad (2.15a)$$

$$E_\phi^{inc} = \beta_{21}(J_t;\mu_1,\varepsilon_1,\mu_2=0,\varepsilon_2=\infty) + \beta_{22}(J_\phi;\mu_1,\varepsilon_1,\mu_2=0,\varepsilon_2=\infty) \quad . \quad (2.15b)$$

For a numerical solution of (2.9) or (2.15) the generating arc is approximated as a sequence of linear segments as depicted in Fig.(2.2) where the approximation to the generating arc is shown in the plane $\phi = 0$. This segmented generating arc is rotated about the z-axis to obtain an approximation to the surface of the body of revolution. The points t_0, t_1, ..., t_{N+1} specify the end points of the linear segments approximating the generating arc and are written in terms of the coordinates ρ and z. The "half points" $t_{1/2}$, $t_{1 1/2}$, ..., $t_{N+1/2}$ are defined as

$$t_{n-1/2} = \frac{(t_n + t_{n-1})}{2} \quad , \quad 1 \le n \le N+1 . \tag{2.16}$$

The variations of the unknown electric and magnetic currents flowing on the surface are approximated by pulse functions in the t-direction and are expanded in Fourier series in the ϕ-direction. The expansion of the electric current is given by

$$\bar{J}(t',\phi') \simeq \frac{\hat{t}}{2\pi\rho'} \sum_{m=-\infty}^{\infty} \sum_{n=1}^{N} I_t^{mn} P_1^n(t') e^{jm\phi'} + \hat{\phi} \sum_{m=-\infty}^{\infty} \sum_{n=1}^{N+1} J_\phi^{mn} P_2^n(t') e^{jm\phi'} . \tag{2.17a}$$

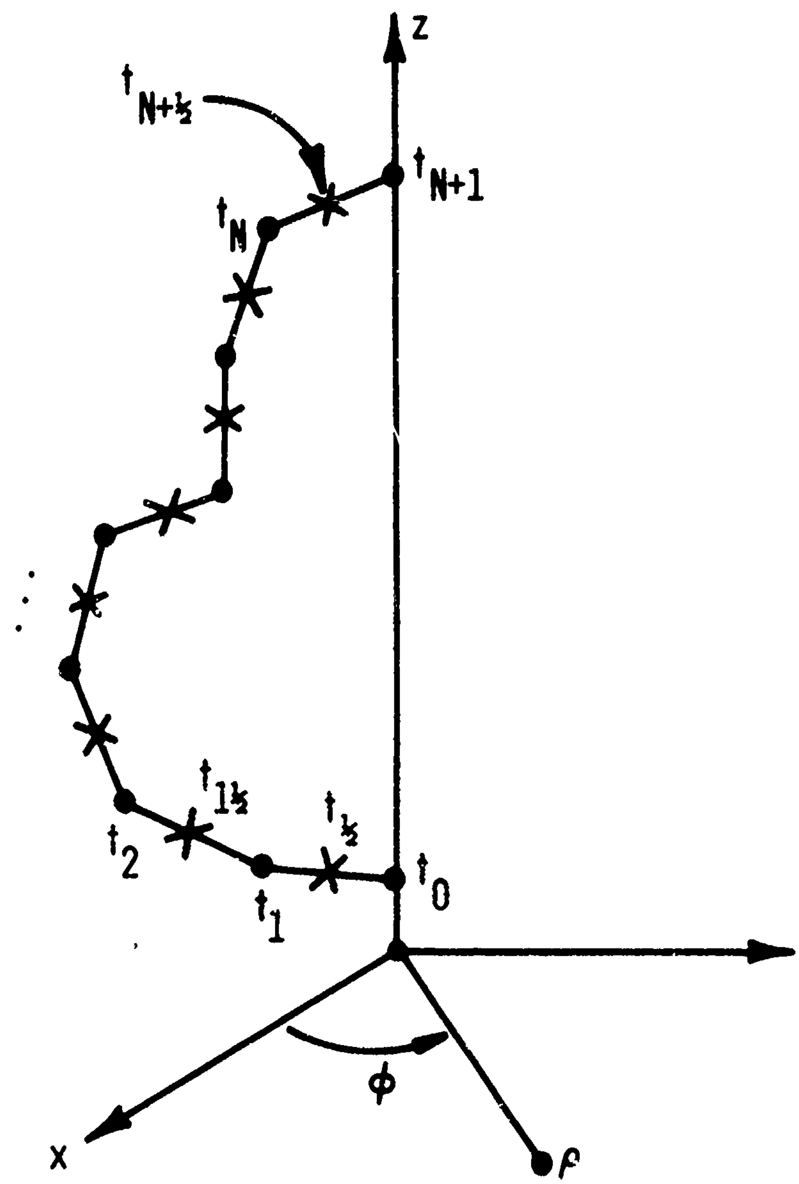

Figure 2.2. Approximation of the generating arc by linear segments.

The derivative of J_t with respect to t, which contributes to the charge, is approximated as

$$\frac{\partial}{\partial t'}[\rho' J_t(t',\phi')] \simeq \frac{1}{2\pi} \sum_{m=-\infty}^{\infty} \sum_{n=1}^{N+1} \left\{ \frac{I_t^{mn} - I_t^{m,n-1}}{\Delta t_n} \right\} P_2^n(t') e^{jm\phi'} \quad . \tag{2.17b}$$

In the preceding,

$$P_1^n(t') = \begin{cases} 1 , & t_{n-\frac{1}{2}} \leq t' \leq t_{n+\frac{1}{2}} \\ 0 , & \text{otherwise} \end{cases} \quad , \tag{2.18a}$$

$$P_2^n(t') = \begin{cases} 1 , & t_{n-1} \leq t' \leq t_n \\ 0 , & \text{otherwise} \end{cases} \quad , \tag{2.18b}$$

$$\begin{aligned} \Delta t_n &= |t_n - t_{n-1}| \\ &= [(\rho_n - \rho_{n-1})^2 + (z_n - z_{n-1})^2]^{\frac{1}{2}} \quad , \end{aligned} \tag{2.19}$$

and I_t^{mn} is the coefficient of the "total" current flowing in the t-direction as defined by

$$I_t(t',\phi') = 2\pi\rho' J_t(t',\phi') \quad . \tag{2.20}$$

This "total" current has no physical meaning (except for $m = 0$; for $m \neq 0$, the integrated current vanishes) but, as a mathematical concept, its use is advantageous in the

solution procedure. In (2.17b) it is assumed that

$$I_t^{m,0} \equiv I_t^{m,N+1} \equiv 0 \quad .$$

The magnetic current $\overline{M}(t',\phi')$ and its derivative, $\frac{\partial}{\partial t'}[\rho' M_t(t',\phi')]$, are defined similarly with $K_t^{mn} = 2\pi\rho' M_t^{mn}$ and M_ϕ^{mn} replacing I_t^{mn} and J_ϕ^{mn}, respectively.

The current expansion scheme (2.17) has a number of advantages. The first, and most obvious, is that the Fourier components of the current can be decoupled; thus one can solve for each unknown Fourier component of the current distribution independently. The remaining advantages arise from the use of the staggered pulse basis sets indicated by (2.18). To further illustrate some of these advantages we consider the case of a perfectly conducting cylinder (Fig. 2.3) which has one closed end and one open end. Note that the "total" current flowing in the t-direction (I_t) is zero for all Fourier components at $t = 0$ and $t = b$ [6,7] and should therefore be well represented in the neighborhood of these two points by half pulses with a zero coefficient, as in the cases of the TE strip and the rectangular bent plate described in other works [8,9]. On the other hand, the current density in the ϕ-direction (J_ϕ) approaches zero ($m \neq \pm 1$) or a constant ($m = \pm 1$) as $t \to 0$ [6,7]

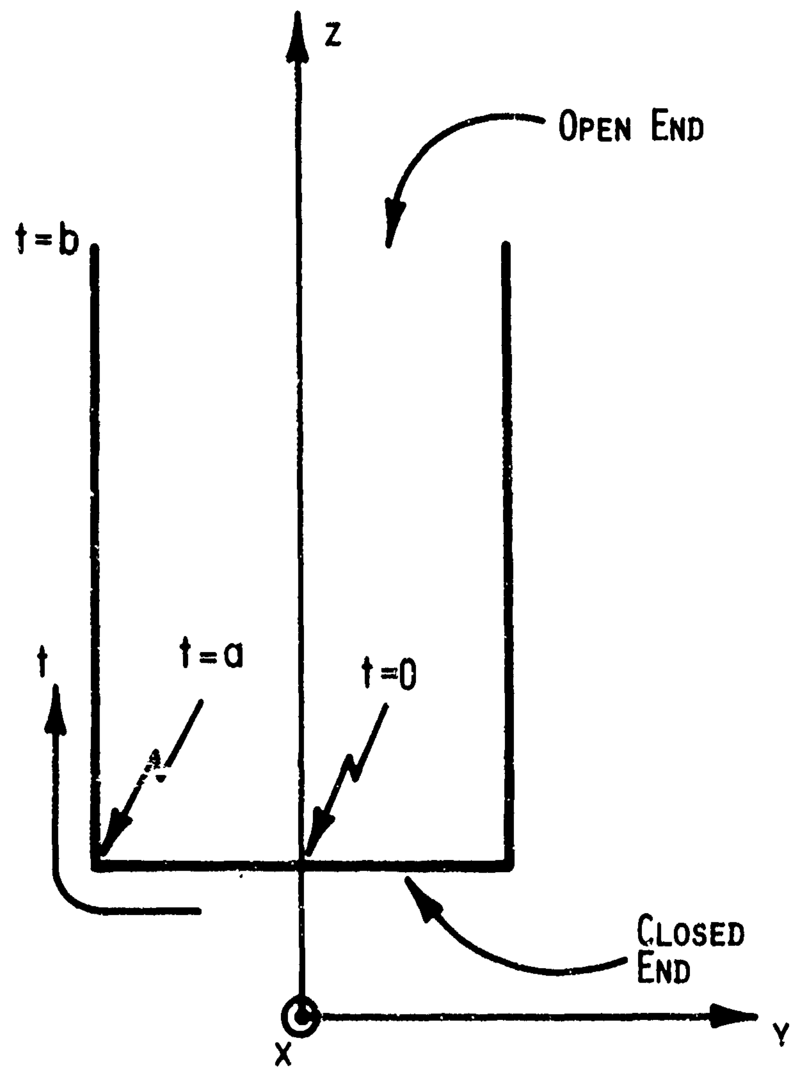

Figure 2.3. Cross section of a perfectly conducting cylinder with one open end.

and is always singular at $t = b$, as required by the so-called edge condition [10]. It has been demonstrated in the previous works [8,9] that the staggered subdomain scheme, such as (2.17) and (2.18), employing full pulses to represent J_ϕ adjacent to the points $t = 0$ and $t = b$, can accurately model the singular currents near edges and should clearly model a finite current as well. The full pulse, however, may not model a current which approaches zero well, but in view of the fact that such a situation occurs only at points where the surface intersects the axis of the body of revolution ($\rho = 0$) and that the current moment represented by the pulse is therefore relatively small, this deficiency should not significantly affect integral functionals of the current such as radar cross section, etc. Thus the staggered pulse-Fourier series basis set enjoys the same advantages described for the staggered subdomain scheme on the rectangular bent plate, namely, the current expansion ensures that t-directed components of current vanish at knife-like edges and are continuous at structure edges (sharp bends), and that the ϕ-directed current and the charge, both of which are singular at an edge, are allowed to vary independently on opposite sides of an edge. The basis set (2.17) therefore allows one to model both open and closed bodies, and bodies with sharp edges, with no special procedures required

at edges or points where the surface intersects the body axis. Of course, a *dielectric* body has no knife-like edges, but it may have structure edges at which, depending on the parameters of the medium, it may be necessary to represent currents which are singular [11]. The basis set (2.17), which is used for both the electric and magnetic currents, provides accurate modeling in the vicinity of such edges.

We next define the testing functions

$$T_1^{pq}(t,\phi) = P_1^q(t)e^{-jp\phi} \qquad (2.21a)$$

$$T_2^{pq}(t,\phi) = P_2^q(t)e^{-jp\phi} \quad . \qquad (2.21b)$$

Eqs. (2.9a) and (2.9c) are tested with (2.21a), while (2.9b) and (2.9d) are tested with (2.21b). The testing procedure results in two surface integrations being required for each β_{ij}. However, we note that the current has already been expanded as a Fourier series in the variable ϕ'. Likewise the various kernel terms appearing in (2.12) may be expanded in Fourier series of the form

$$\frac{e^{-jk_iR_0}}{R_0} = \frac{1}{2\pi}\sum_{m=-\infty}^{\infty} G_{i_m} e^{jm(\phi - \phi')} \quad ,$$

$$i = 1,2 \; ,$$

and

$$\frac{1}{R_0}\frac{d}{dR_0}\frac{e^{-jk_iR_0}}{R_0} = \frac{1}{2\pi}\sum_{m=-\infty}^{\infty} G'_{i_m} e^{jm(\phi-\phi')} \quad ,$$

$$i = 1,2 \; ,$$

with Fourier coefficients

$$G_{i_m} = \int_{-\pi}^{\pi} \frac{e^{-jk_iR_0}}{R_0} \cos(m\xi)\, d\xi \qquad (2.22a)\;.$$

$$G'_{i_m} = \int_{-\pi}^{\pi} \frac{1}{R_0}\frac{d}{dR_0}\left\{\frac{e^{-jk_iR_0}}{R_0}\right\}\cos(m\xi)\, d\xi \quad , \qquad (2.22b)$$

where

$$R_0 = [\rho^2 + \rho'^2 - 2\rho\rho'\cos\xi + (z-z')^2]^{\frac{1}{2}} \quad . \qquad (2.22c)$$

Testing with the testing functions (2.21) permits all integrations in the variables ϕ and ϕ' to be carried out analytically, simultaneously decoupling the equations with respect to their dependence on the indices p and m. The double integral in the t-direction is reduced to a single integral via appropriate approximations (such as used in [8,9]). We also note that derivatives with respect to ϕ and ϕ' can be performed analytically during the testing procedure. By means of such approximations and procedures, one obtains

expressions for the elements $\beta^{qn}_{ij_m}$ of the generalized impedance matrix, where the subscript m refers to the Fourier coefficient index and the superscripts q and n refer to the indices of the field point and the current source, respectively. For convenience in writing these expressions, we first define two frequently used integral functions:

$$\psi_i(t_1,t_2;t_q,m) = \int_{t_1}^{t_2} G_{i_m}(t_q,t')\, dt' \qquad (2.23a)$$

$$\psi_i^{\rho}(t_1,t_2;t_q,m) = \int_{t_1}^{t_2} G_{i_m}(t_q,t')\rho'\, dt' , \qquad (2.23b)$$

where G_{i_m} is defined by (2.22). We also introduce the auxiliary "weight" functions arising from testing in the t-direction:

$$\chi_s(\Delta t_q,\gamma_q) = \frac{\Delta t_{q+1}\sin\gamma_{q+1} + \Delta t_q \sin\gamma_q}{2} \qquad (2.24a)$$

$$\chi_c(\Delta t_q,\gamma_q) = \frac{\Delta t_{q+1}\cos\gamma_{q+1} + \Delta t_q \cos\gamma_q}{2} . \qquad (2.24b)$$

The expressions for the elements of the generalized impedance matrix are given on the following pages:

$$
\begin{aligned}
\beta_{11_m}^{qn} = {} & \frac{j\omega}{8\pi}\sin\gamma_n \chi_s(\Delta t_q,\gamma_q)[\mu_1\psi_1(t_{n-\frac{1}{2}},t_n;t_q,m+1) + \mu_2\psi_2(t_{n-\frac{1}{2}},t_n;t_q,m+1) \\
& \qquad + \mu_1\psi_1(t_{n-\frac{1}{2}},t_n;t_q,m-1) + \mu_2\psi_2(t_{n-\frac{1}{2}},t_n;t_q,m-1)] \\
& + \frac{j\omega}{8\pi}\sin\gamma_{n+1}\chi_s(\Delta t_q,\gamma_q)[\mu_1\psi_1(t_n,t_{n+\frac{1}{2}};t_q,m+1) + \mu_2\psi_2(t_n,t_{n+\frac{1}{2}};t_q,m+1) \\
& \qquad + \mu_1\psi_1(t_n,t_{n+\frac{1}{2}};t_q,m-1) + \mu_2\psi_2(t_n,t_{n+\frac{1}{2}};t_q,m-1)] \\
& + \frac{j\omega}{4\pi}\cos\gamma_n\chi_c(\Delta t_q,\gamma_q)[\mu_1\psi_1(t_{n-\frac{1}{2}},t_n;t_q,m) + \mu_2\psi_2(t_{n-\frac{1}{2}},t_n;t_q,m)] \\
& + \frac{j\omega}{4\pi}\cos\gamma_{n+1}\chi_c(\Delta t_q,\gamma_q)[\mu_1\psi_1(t_n,t_{n+\frac{1}{2}};t_q,m) + \mu_2\psi_2(t_n,t_{n+\frac{1}{2}};t_q,m)] \\
& + \frac{j}{4\pi\omega\Delta t_n}[\frac{1}{\varepsilon_1}\psi_1(t_{n-1},t_n;t_{q+\frac{1}{2}},m) + \frac{1}{\varepsilon_2}\psi_2(t_{n-1},t_n;t_{q+\frac{1}{2}},m) - \frac{1}{\varepsilon_1}\psi_1(t_{n-1},t_n;t_{q-\frac{1}{2}},m) \\
& \qquad - \frac{1}{\varepsilon_2}\psi_2(t_{n-1},t_n;t_{q-\frac{1}{2}},m)] \\
& - \frac{j}{4\pi\omega\Delta t_{n+1}}[\frac{1}{\varepsilon_1}\psi_1(t_n,t_{n+1};t_{q+\frac{1}{2}},m) + \frac{1}{\varepsilon_2}\psi_2(t_n,t_{n+1};t_{q+\frac{1}{2}},m) - \frac{1}{\varepsilon_1}\psi_1(t_n,t_{n+1};t_{q-\frac{1}{2}},m) \\
& \qquad - \frac{1}{\varepsilon_2}\psi_2(t_n,t_{n+1};t_{q-\frac{1}{2}},m)] \quad , \qquad (2.25a)
\end{aligned}
$$

$$
\begin{aligned}
q &= 1,2,\ldots,N \ , \\
n &= 1,2,\ldots,N \ ,
\end{aligned}
$$

$$
\begin{aligned}
\beta_{12_m}^{qn} = & -\tfrac{1}{2}\omega\chi_s(\Delta t_q,\gamma_q)[\mu_1\psi_1^{\rho}(t_{n-1},t_n;t_q,m+1) + \mu_2\psi_2^{\rho}(t_{n-1},t_n;t_q,m+1) \\
& \qquad - \mu_1\psi_1^{\rho}(t_{n-1},t_n;t_q,m-1) - \mu_2\psi_2^{\rho}(t_{n-1},t_n;t_q,m-1)] \\
& - \frac{m}{2\omega}[\frac{1}{\varepsilon_1}\psi_1(t_{n-1},t_n;t_{q+\frac{1}{2}},m) + \frac{1}{\varepsilon_2}\psi_2(t_{n-1},t_n;t_{q+\frac{1}{2}},m) - \frac{1}{\varepsilon_1}\psi_1(t_{n-1},t_n;t_{q-\frac{1}{2}},m) \\
& \quad - \frac{1}{\varepsilon_2}\psi_2(t_{n-1},t_n;t_{q-\frac{1}{2}},m)] \ ,
\end{aligned} \tag{2.25b}
$$

$$
q = 1,2,\ldots,N \ , \quad n = 1,2,\ldots,N+1 \ ,
$$

$$
\begin{aligned}
\beta_{21_m}^{qn} = & \frac{\omega}{8\pi}\Delta t_q \sin\gamma_n[\mu_1\psi_1(t_{n-\frac{1}{2}},t_n;t_{q-\frac{1}{2}},m+1) + \mu_2\psi_2(t_{n-\frac{1}{2}},t_n;t_{q-\frac{1}{2}},m+1) \\
& \qquad - \mu_1\psi_1(t_{n-\frac{1}{2}},t_n;t_{q-\frac{1}{2}},m-1) - \mu_2\psi_2(t_{n-\frac{1}{2}},t_n;t_{q-\frac{1}{2}},m-1)] \\
& + \frac{\omega}{8\pi}\Delta t_q \sin\gamma_{n+1}[\mu_1\psi_1(t_n,t_{n+\frac{1}{2}};t_{q-\frac{1}{2}},m+1) + \mu_2\psi_2(t_n,t_{n+\frac{1}{2}};t_{q-\frac{1}{2}},m+1) \\
& \qquad - \mu_1\psi_1(t_n,t_{n+\frac{1}{2}};t_{q-\frac{1}{2}},m-1) - \mu_2\psi_2(t_n,t_{n+\frac{1}{2}};t_{q-\frac{1}{2}},m-1)] \\
& - \frac{m\Delta t_q}{4\pi\omega\rho_{q-\frac{1}{2}}}[\frac{1}{\varepsilon_1}\psi_1(t_{n-1},t_n;t_{q-\frac{1}{2}},m) + \frac{1}{\varepsilon_2}\psi_2(t_{n-1},t_n;t_{q-\frac{1}{2}},m)] \\
& + \frac{m\Delta t_q}{4\pi\omega\rho_{q-\frac{1}{2}}}[\frac{1}{\varepsilon_1}\psi_1(t_n,t_{n+1};t_{q-\frac{1}{2}},m) + \frac{1}{\varepsilon_2}\psi_2(t_n,t_{n+1};t_{q-\frac{1}{2}},m)] \ ,
\end{aligned} \tag{2.25c}
$$

$$
q = 1,2,\ldots,N+1 \ , \quad n = 1,2,\ldots,N \ ,
$$

$$\beta_{22_m}^{qn} = \tfrac{1}{2}j\omega\Delta t_q[\mu_1\psi_1^\rho(t_{n-1},t_n;t_{q-\frac{1}{2}},m+1) + \mu_2\psi_2^\rho(t_{n-1},t_n;t_{q-\frac{1}{2}},m+1)$$

$$+ \mu_1\psi_1^\rho(t_{n-1},t_n;t_{q-\frac{1}{2}},m-1) + \mu_2\psi_2^\rho(t_{n-1},t_n;t_{q-\frac{1}{2}},m-1)]$$

$$- \frac{jm^2\Delta t_q}{2\omega\rho_{q-\frac{1}{2}}}[\frac{1}{\varepsilon_1}\psi_1(t_{n-1},t_n;t_{q-\frac{1}{2}},m) + \frac{1}{\varepsilon_2}\psi_2(t_{n-1},t_n;t_{q-\frac{1}{2}},m)] \; , \qquad (2.25d)$$

$$q = 1,2,\ldots,N+1 \; ,$$
$$n = 1,2,\ldots,N+1 \; ,$$

$$\beta_{13_m}^{qn} = -j\frac{\cos\gamma_n}{4\pi}\chi_s(\Delta t_q,\gamma_q)U_3^\rho(t_{n-\frac{1}{2}},t_n;t_q,m) - j\frac{\cos\gamma_{n+1}}{4\pi}\chi_s(\Delta t_q,\gamma_q)U_3^\rho(t_n,t_{n+\frac{1}{2}};t_q,m)$$

$$+ j\rho_q\frac{\sin\gamma_n}{4\pi}\chi_c(\Delta t_q,\gamma_q)U_0(t_{n-\frac{1}{2}},t_n;t_q,m) + j\rho_q\frac{\sin\gamma_{n+1}}{4\pi}\chi_c(\Delta t_q,\gamma_q)U_0(t_n,t_{n+\frac{1}{2}};t_q,m)$$

$$- j\frac{\sin\gamma_n}{4\pi}\chi_s(\Delta t_q,\gamma_q)U_1(t_{n-\frac{1}{2}},t_n;t_q,m) - j\frac{\sin\gamma_{n+1}}{4\pi}\chi_s(\Delta t_q,\gamma_q)U_1(t_n,t_{n+\frac{1}{2}};t_q,m) \; ,$$

$$q = 1,2,\ldots,N \; , \qquad (2.25e)$$
$$n = 1,2,\ldots,N \; ,$$

$$\beta_{14_m}^{qn} = \tfrac{1}{2}\chi_c(\Delta t_q,\gamma_q)U_4^\rho(t_{n-1},t_n;t_q,m) - \rho_q\chi_c(\Delta t_q,\gamma_q)U_5^\rho(t_{n-1},t_n;t_q,m)$$

$$- \tfrac{1}{2}\chi_s(\Delta t_q,\gamma_q)U_2^\rho(t_{n-1},t_n;t_q,m) \; , \qquad q = 1,2,\ldots,N \; , \qquad (2.25f)$$
$$n = 1,2,\ldots,N+1 \; ,$$

$$\beta_{23_m}^{qn} = -\frac{\rho_{q-\frac{1}{2}}}{2\pi}\cos\gamma_n \Delta t_q U_5(t_{n-\frac{1}{2}},t_n;t_{q-\frac{1}{2}},m) - \frac{\rho_{q-\frac{1}{2}}}{2\pi}\cos\gamma_{n+1}\Delta t_q U_5(t_n,t_{n+\frac{1}{2}};t_{q-\frac{1}{2}},m)$$

$$-\frac{1}{4\pi}\cos\gamma_n \Delta t_q U_6(t_{n-\frac{1}{2}},t_n;t_{q-\frac{1}{2}},m) - \frac{1}{4\pi}\cos\gamma_{n+1}\Delta t_q U_6(t_n,t_{n+\frac{1}{2}};t_{q-\frac{1}{2}},m)$$

$$+\frac{1}{4\pi}\sin\gamma_n \Delta t_q U_2(t_{n-\frac{1}{2}},t_n;t_{q-\frac{1}{2}},m) + \frac{1}{4\pi}\sin\gamma_{n+1}\Delta t_q U_2(t_n,t_{n+\frac{1}{2}};t_{q-\frac{1}{2}},m) \quad , \qquad (2.25g)$$

$$q = 1,2,\ldots,N+1 \; ,$$
$$n = 1,2,\ldots,N \; ,$$

$$\beta_{24_m}^{qn} = -\tfrac{1}{2} j \Delta t_q U_1^{\rho}(t_{n-1},t_n;t_{q-\frac{1}{2}},m) \quad . \qquad (2.25h)$$

$$q = 1,2,\ldots,N+1 \; ,$$
$$n = 1,2,\ldots,N+1 \; .$$

The U's in (2.25) are integral functions defined by

$$U_0(t_1,t_2;t_q,m) = \int_{t_1}^{t_2}\int_{-\pi}^{\pi} \frac{\sin(m\xi)\sin\xi}{R_0}\frac{dG}{dR_0}\,d\xi dt' \tag{2.26a}$$

$$U_1(t_1,t_2;t_q,m) = \int_{t_1}^{t_2}\int_{-\pi}^{\pi} \frac{(z-z')}{R_0}\sin(m\xi)\sin\xi\,\frac{dG}{dR_0}\,d\xi dt' \tag{2.26b}$$

$$U_1^{\rho}(t_1,t_2;t_q,m) = \int_{t_1}^{t_2}\int_{-\pi}^{\pi} \frac{(z-z')}{R_0}\sin(m\xi)\sin\xi\,\frac{dG}{dR_0}\,\rho' d\xi dt' \tag{2.26c}$$

$$U_2(t_1,t_2;t_q,m) = \int_{t_1}^{t_2}\int_{-\pi}^{\pi} \frac{(z-z')}{R_0}\cos(m\xi)\cos\xi\,\frac{dG}{dR_0}\,d\xi dt' \tag{2.26d}$$

$$U_2^{\rho}(t_1,t_2;t_q,m) = \int_{t_1}^{t_2}\int_{-\pi}^{\pi} \frac{(z-z')}{R_0}\cos(m\xi)\cos\xi\,\frac{dG}{dR_0}\,\rho' d\xi dt' \tag{2.26e}$$

$$U_3^{\rho}(t_1,t_2;t_q,m) = \int_{t_1}^{t_2}\int_{-\pi}^{\pi} \frac{\rho'}{R_0}\sin(m\xi)\sin\xi\,\frac{dG}{dR_0}\,d\xi dt' \tag{2.26f}$$

$$U_4^{\rho}(t_1,t_2;t_q,m) = \int_{t_1}^{t_2}\int_{-\pi}^{\pi} \frac{(\rho-\rho')}{R_0}\cos(m\xi)\,\frac{dG}{dR_0}\,\rho' d\xi dt' \tag{2.26g}$$

$$U_5(t_1,t_2;t_q,m) = \int_{t_1}^{t_2}\int_{-\pi}^{\pi} \frac{\cos(m\xi)}{R_0}\sin^2(\xi/2)\ \frac{dG}{dR_0}\ d\xi dt' \qquad (2.26h)$$

$$U_5^{\rho}(t_1,t_2;t_q,m) = \int_{t_1}^{t_2}\int_{-\pi}^{\pi} \frac{\cos(m\xi)}{R_0}\sin^2(\xi/2)\ \frac{dG}{dR_0}\ \rho' d\xi dt' \qquad (2.26i)$$

$$U_6(t_1,t_2;t_q,m) = \int_{t_1}^{t_2}\int_{-\pi}^{\pi} \frac{(\rho - \rho')}{R_0}\cos(m\xi)\cos\xi\ \frac{dG}{dR_0}\ d\xi dt', \qquad (2.26j)$$

where R_0 is defined in (2.22c) and

$$G = \frac{e^{-jk_1R_0}}{R_0} + \frac{e^{-jk_2R_0}}{R_0} \quad . \qquad (2.27)$$

The integrals defined by the U's are actually recombinations of expressions involving the Fourier coefficients (2.22b) and are expressed in the manner shown to facilitate the singularity analysis of self terms, which appears in Appendix A. We have now completely defined the elements of the impedance matrix corresponding to the operators appearing in (2.9a) and (2.9b); the matrix operators arising from (2.9c) and (2.9d) are obtained from the functional relationships (2.14). One should note that many of the integrals which must be computed in the calculations of the elements (2.25)

appear in several places and hence can be used several times to increase efficiency.

The incident plane wave fields are expressed as

$$\overline{E}^{inc} = (E_\theta^i \hat{\theta}^i + E_\phi^i \hat{\phi}^i) e^{-jk_1 \hat{n} \cdot \overline{r}} \tag{2.28a}$$

$$\overline{H}^{inc} = \frac{1}{\eta}(E_\phi^i \hat{\theta}^i - E_\theta^i \hat{\phi}^i) e^{-jk_1 \hat{n} \cdot \overline{r}} \quad , \tag{2.28b}$$

where

$$\hat{\theta}^i = \cos\theta^i \cos\phi^i \, \hat{x} + \cos\theta^i \sin\phi^i \, \hat{y} - \sin\theta^i \, \hat{z} \tag{2.29a}$$

$$\hat{\phi}^i = -\sin\phi^i \, \hat{x} + \cos\phi^i \, \hat{y} \tag{2.29b}$$

$$\hat{n} = -\sin\theta^i \cos\phi^i \, \hat{x} - \sin\theta^i \sin\phi^i \, \hat{y} - \cos\theta^i \, \hat{z} \tag{2.29c}$$

$$\overline{r} = \rho\cos\phi \, \hat{x} + \rho\sin\phi \, \hat{y} + z\hat{z} \quad , \tag{2.29d}$$

and η is the free space impedance. Note that $\hat{n}$ is in the direction of propagation. The elements of the forcing function vector are thus determined by finding the components tangential to the surface and testing with (2.21). With reference to (2.9), these elements are given by the expressions on the following page, where $J_m(u)$ is the Bessel function of the first kind of order m. With regard to the expressions (2.30), we comment that, with no loss in generality, one may set $\phi^i = 0$.

$$E^{inc^q}_{t_m} = \{E^i_\theta \cos\theta^i \chi_s(\Delta t_q,\gamma_q)[j^{m-1}J_{m-1}(k_1\rho_q\sin\theta^i) + j^{m+1}J_{m+1}(k_1\rho_q\sin\theta^i)]$$

$$- 2E^i_\theta \sin\theta^i \chi_c(\Delta t_q,\gamma_q) j^m J_m(k_1\rho_q\sin\theta^i)$$

$$- E^i_\phi \chi_s(\Delta t_q,\gamma_q) j^m[J_{m-1}(k_1\rho_q\sin\theta^i) + J_{m+1}(k_1\rho_q\sin\theta^i)]\}\pi e^{jk_1 z_q\cos\theta^i} e^{-jm\phi^i},$$

$$q = 1,2,\ldots,N, \qquad (2.30a)$$

$$E^{inc^q}_{\phi_m} = \{E^i_\theta j^m \cos\theta^i [J_{m-1}(k_1\rho_{q-\frac{1}{2}}\sin\theta^i) + J_{m+1}(k_1\rho_{q-\frac{1}{2}}\sin\theta^i)]$$

$$+ E^i_\phi [j^{m-1}J_{m-1}(k_1\rho_{q-\frac{1}{2}}\sin\theta^i) + j^{m+1}J_{m+1}(k_1\rho_{q-\frac{1}{2}}\sin\theta^i)]\}\pi\Delta t_q e^{jk_1 z_{q-\frac{1}{2}}\cos\theta^i} e^{-jm\phi^i},$$

$$q = 1,2,\ldots,N+1, \qquad (2.30b)$$

$$H^{inc^q}_{t_m} = \{E^i_\phi \cos\theta^i \chi_s(\Delta t_q,\gamma_q)[j^{m-1}J_{m-1}(k_1\rho_q\sin\theta^i) + j^{m+1}J_{m+1}(k_1\rho_q\sin\theta^i)]$$

$$- 2E^i_\phi \sin\theta^i \chi_c(\Delta t_q,\gamma_q) j^m J_m(k_1\rho_q\sin\theta^i)$$

$$+ E^i_\theta \chi_s(\Delta t_q,\gamma_q) j^m[J_{m-1}(k_1\rho_q\sin\theta^i) + J_{m+1}(k_1\rho_q\sin\theta^i)]\}\frac{\pi}{\eta} e^{jk_1 z_q\cos\theta^i} e^{-jm\phi^i},$$

$$q = 1,2,\ldots,N, \qquad (2.30c)$$

$$H^{inc^q}_{\phi_m} = \{E^i_\phi j^m \cos\theta^i [J_{m-1}(k_1\rho_{q-\frac{1}{2}}\sin\theta^i) + J_{m+1}(k_1\rho_{q-\frac{1}{2}}\sin\theta^i)]$$

$$- E^i_\theta [j^{m-1}J_{m-1}(k_1\rho_{q-\frac{1}{2}}\sin\theta^i) + j^{m+1}J_{m+1}(k_1\rho_{q-\frac{1}{2}}\sin\theta^i)]\}\frac{\pi\Delta t_q}{\eta} e^{jk_1 z_{q-\frac{1}{2}}\cos\theta^i} e^{-jm\phi^i},$$

$$q = 1,2,\ldots,N+1. \qquad (2.30d)$$

A full solution for the currents on the body of revolution consists, in general, of an infinite number of Fourier components. In practice, however, the series is truncated such that $-M \leq m \leq M$, where M is the maximum number of positive Fourier components to be calculated. Furthermore, it can be shown that the current solution for $-m$ is related to the solution for $+m$, so that it is only necessary to compute the impedance matrix and drive vector for $m = 0, 1, 2, \ldots, M$. This relationship is determined by comparing the signs of the impedance matrix and a decomposed drive vector for the positive and negative Fourier component indices. If one has available the impedance submatrices β_{ij_m} for positive m, then the impedance matrix for the corresponding negative Fourier component, $-m$, is given by

$$\begin{bmatrix} \beta_{11_m} & -\beta_{12_m} & -\beta_{13_m} & \beta_{14_m} \\ -\beta_{21_m} & \beta_{22_m} & \beta_{23_m} & -\beta_{24_m} \\ -\beta_{31_m} & \beta_{32_m} & \beta_{33_m} & -\beta_{34_m} \\ \beta_{41_m} & -\beta_{42_m} & -\beta_{43_m} & \beta_{44_m} \end{bmatrix} \quad . \tag{2.31}$$

The submatrices constituting the drive vector can be resolved as follows:

$$
\begin{bmatrix} E^{inc}_{t_m} \\ E^{inc}_{\phi_m} \\ H^{inc}_{t_m} \\ H^{inc}_{\phi_m} \end{bmatrix} = \begin{bmatrix} \alpha^{\theta}_{1_m} \\ \alpha^{\theta}_{2_m} \\ \alpha^{\theta}_{3_m} \\ \alpha^{\theta}_{4_m} \end{bmatrix} + \begin{bmatrix} \alpha^{\phi}_{1_m} \\ \alpha^{\phi}_{2_m} \\ \alpha^{\phi}_{3_m} \\ \alpha^{\phi}_{4_m} \end{bmatrix} , \qquad (2.32)
$$

where the superscripts θ and ϕ refer to the components of the incident field arising from E^i_θ and E^i_ϕ, respectively. We then find that

$$
\begin{bmatrix} \alpha^{\theta}_{1_{-m}} \\ \alpha^{\theta}_{2_{-m}} \\ \alpha^{\theta}_{3_{-m}} \\ \alpha^{\theta}_{4_{-m}} \end{bmatrix} = \begin{bmatrix} \alpha^{\theta}_{1_m} \\ -\alpha^{\theta}_{2_m} \\ -\alpha^{\theta}_{3_m} \\ \alpha^{\theta}_{4_m} \end{bmatrix} \qquad (2.33a)
$$

and that

$$
\begin{bmatrix} \alpha^{\phi}_{1_{-m}} \\ \alpha^{\phi}_{2_{-m}} \\ \alpha^{\phi}_{3_{-m}} \\ \alpha^{\phi}_{4_{-m}} \end{bmatrix} = \begin{bmatrix} -\alpha^{\phi}_{1_m} \\ \alpha^{\phi}_{2_m} \\ \alpha^{\phi}_{3_m} \\ -\alpha^{\phi}_{4_m} \end{bmatrix} . \qquad (2.33b)
$$

Thus the decomposed negative Fourier component solution vectors are related to the corresponding positive component solution vectors by

$$\begin{bmatrix} I^{\theta}_{t_{-m}} \\ J^{\theta}_{\phi_{-m}} \\ K^{\theta}_{t_{-m}} \\ M^{\theta}_{\phi_{-m}} \end{bmatrix} = \begin{bmatrix} I^{\theta}_{t_{m}} \\ -J^{\theta}_{\phi_{m}} \\ -K^{\theta}_{t_{m}} \\ M^{\theta}_{\phi_{m}} \end{bmatrix} \tag{2.34a}$$

$$\begin{bmatrix} I^{\phi}_{t_{-m}} \\ J^{\phi}_{\phi_{-m}} \\ K^{\phi}_{t_{-m}} \\ M^{\phi}_{\phi_{-m}} \end{bmatrix} = \begin{bmatrix} -I^{\phi}_{t_{m}} \\ J^{\phi}_{\phi_{m}} \\ K^{\phi}_{t_{m}} \\ -M^{\phi}_{\phi_{m}} \end{bmatrix} , \tag{2.34b}$$

which may be combined to obtain

$$\begin{bmatrix} I_{t_{m}} \\ J_{\phi_{m}} \\ K_{t_{m}} \\ M_{\phi_{m}} \end{bmatrix} = \begin{bmatrix} I^{\theta}_{t_{m}} \\ J^{\theta}_{\phi_{m}} \\ K^{\theta}_{t_{m}} \\ M^{\theta}_{\phi_{m}} \end{bmatrix} + \begin{bmatrix} I^{\phi}_{t_{m}} \\ J^{\phi}_{\phi_{m}} \\ K^{\phi}_{t_{m}} \\ M^{\phi}_{\phi_{m}} \end{bmatrix} \tag{2.35a}$$

$$
\begin{bmatrix} I_{t_{-m}} \\ J_{\phi_{-m}} \\ K_{t_{-m}} \\ M_{\phi_{-m}} \end{bmatrix} = \begin{bmatrix} I^{\theta}_{t_m} \\ -J^{\theta}_{\phi_m} \\ -K^{\theta}_{t_m} \\ M^{\theta}_{\phi_m} \end{bmatrix} + \begin{bmatrix} -I^{\phi}_{t_m} \\ J^{\phi}_{\phi_m} \\ K^{\phi}_{t_m} \\ -M^{\phi}_{\phi_m} \end{bmatrix} . \qquad (2.35b)
$$

Thus Eqs. (2.9) (or (2.14) for a perfect conductor) constitute a linear system of equations for the current coefficients in (2.17). The Fourier components of the current in the problem are decoupled, however, and one may solve for each Fourier component current distribution individually, using the impedance matrix elements given by (2.25) and the elements of the forcing function vector, (2.30). The forcing function vector is then decomposed into two drive vectors as indicated by (2.32) so that one may obtain both the positively and negatively indexed Fourier components of the current distributions simultaneously, through the relationships (2.35) .

2.2 Numerical Results for the Body of Revolution

The numerical procedures described in the previous section have been incorporated into a computer code, herein referred to as "DBR," and numerical results for the surface

currents on several structures have been obtained and compared with other available data. The code is capable of treating both dielectric and perfectly conducting bodies; in the latter case, the bodies may be either open or closed structures.

All of the structures considered in this section are assumed to be illuminated by a plane wave incident along the z-axis with an electric field polarized in the x-direction. Propagation may be in either the positive or negative z-direction. For a field incident along the axis of the body of revolution, only the Fourier components with $e^{\pm j\phi}$ variation ($m = \pm 1$) are excited. Using the relationships (2.35) the surface currents can then be written in the simple form

$$\bar{J}(t,\phi) = J_t^{\theta}(t)[e^{j\phi} + e^{-j\phi}] + J_{\phi}^{\theta}(t)[e^{j\phi} - e^{-j\phi}]$$

$$= J_t(t)\cos\phi + jJ_{\phi}(t)\sin\phi \tag{2.36a}$$

$$\bar{M}(t,\phi) = M_t^{\theta}(t)[e^{j\phi} - e^{-j\phi}] + M_{\phi}^{\theta}(t)[e^{j\phi} + e^{-j\phi}]$$

$$= jM_t(t)\sin\phi + M_{\phi}(t)\cos\phi \quad , \tag{2.36b}$$

where

$$J_p(t) = 2J_p^{\theta}(t)$$

$$M_p(t) = 2M_p^{\theta}(t) \, , \qquad p = t \text{ or } \phi .$$

The surface currents without superscripts in (2.36) therefore represent the variation in the t-direction of the actual surface current density on the body, consisting of a sum over the excited Fourier components and observed in either the $\phi = 0°$ (J_t, M_ϕ) or the $\phi = 90°$ (J_ϕ, M_t) plane. The figures in this section illustrate the t-variation of these actual current densities when viewed in the appropriate plane. We first consider results obtained for dielectric bodies.

In Figs. 2.4 and 2.5 the electric and magnetic surface current distributions, respectively, are illustrated for a dielectric sphere. Results calculated by means of DBR are compared with results obtained by Wu [12]. The radius of the sphere is $k_1 a = 1$ and its relative constitutive parameters are $\mu_r = 1$ and $\varepsilon_r = 4$. The sphere is excited by a plane wave propagating in the positive z-direction. In general, the data obtained with the computer code DBR agree well with that of Wu. One should note, however, that the results calculated by means of DBR have a slight "glitch" near the points where the surface meets the body axis. It is speculated that the rapidly decreasing radius in this region may necessitate using a smoother current basis (e.g., linear) for the total current I_t in the vicinity of the points where the surface meets the body axis, where it is presently

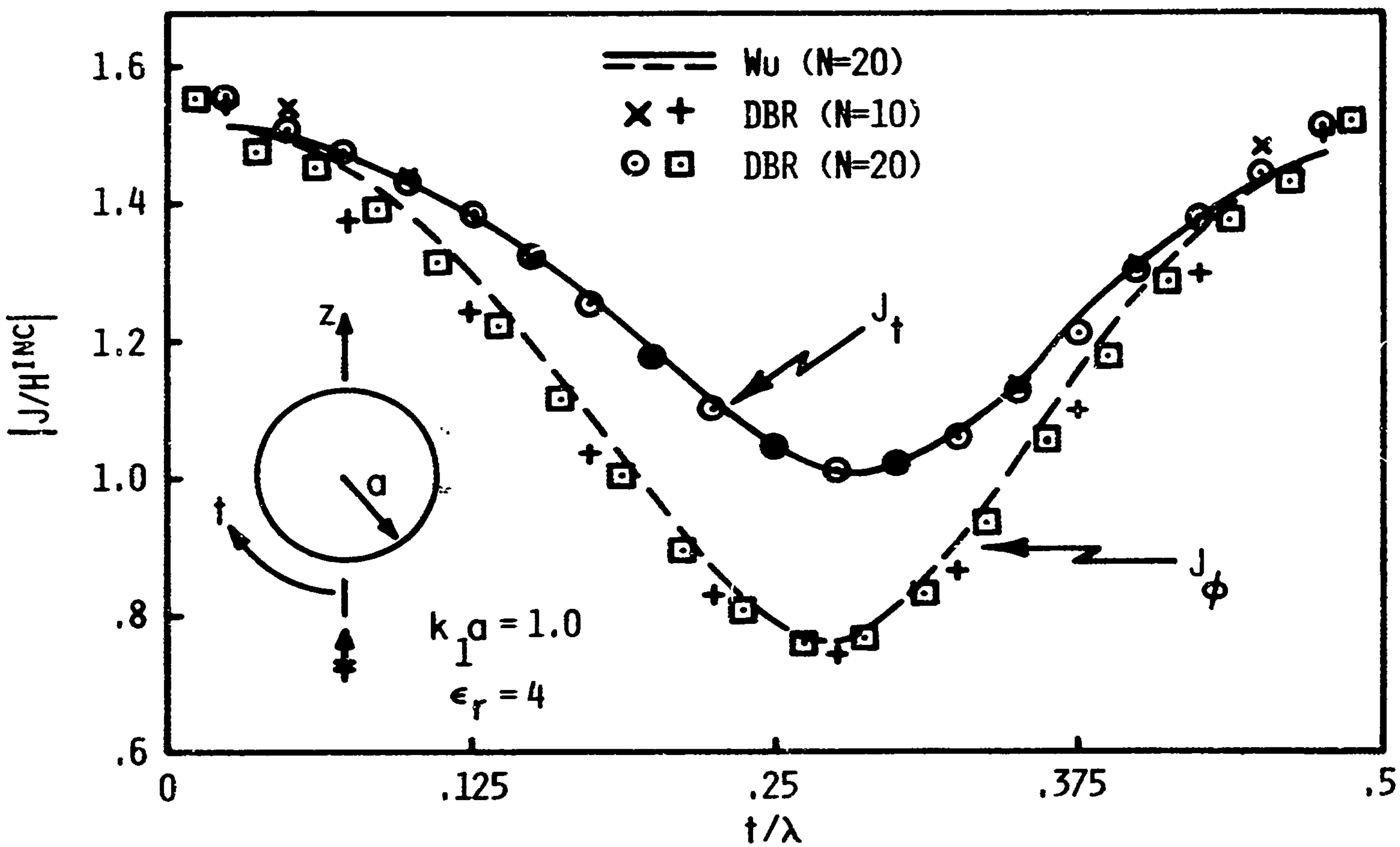

Figure 2.4. Electric surface current distribution on a dielectric sphere illuminated by an axially incident plane wave.

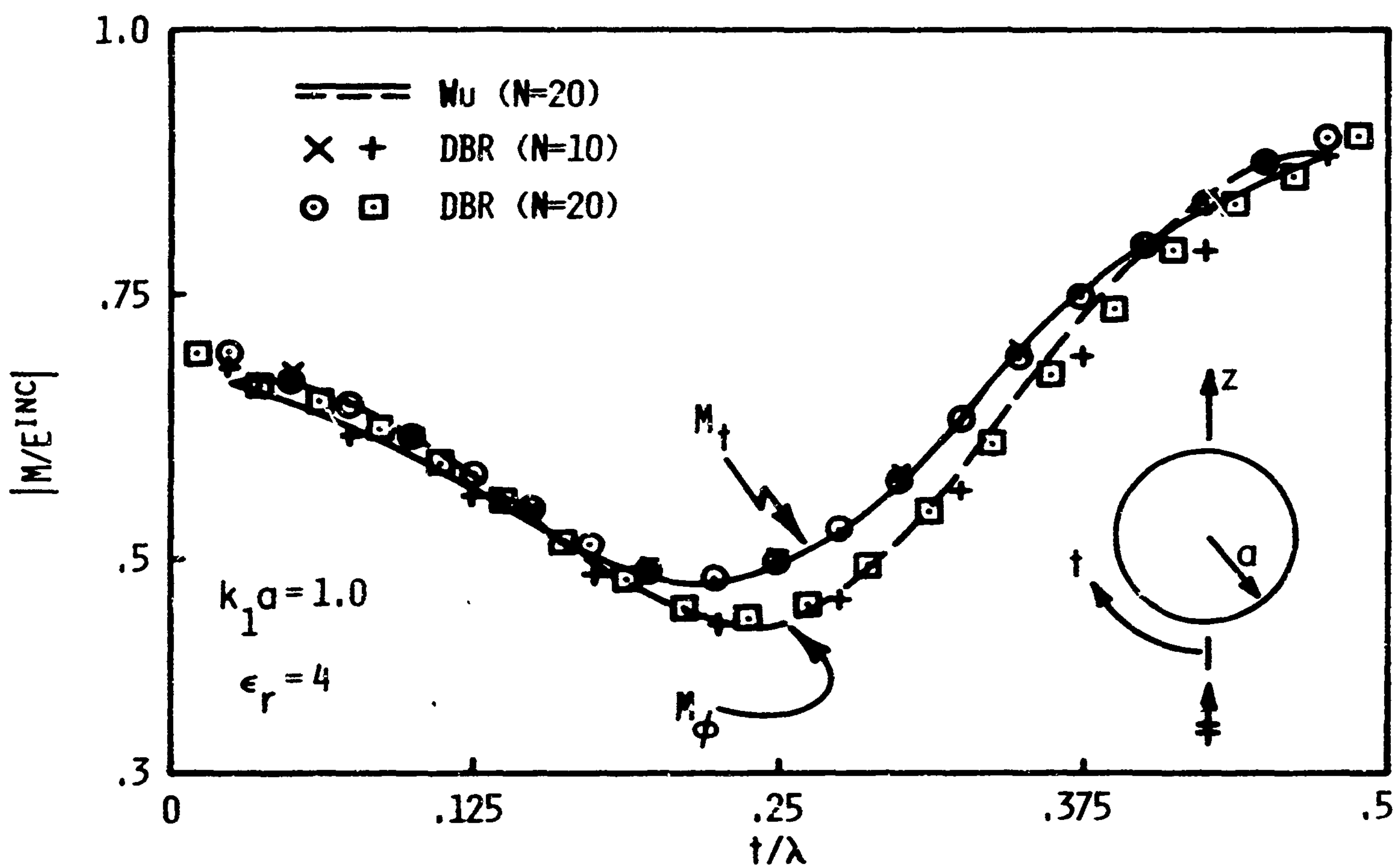

Figure 2.5. Magnetic surface current distribution on a dielectric sphere illuminated by an axially incident plane wave.

modeled by a zero magnitude half pulse. This speculation has not yet been confirmed; however, numerous other attempts to correct the problem have been unsatisfactory. For open conducting bodies where the current density J_t also approaches zero near the ends of the body, no such "glitch" has been detected. The presence of the "glitch" should have a negligible effect on field calculations, however, except possibly very near the points where the surface meets the axis, since the total current moment for the pulses in this region is small.

The electric and magnetic surface current distributions for a dielectric sphere with radius $.2\lambda$ and $\varepsilon_r = 80$ are shown in Figs. 2.6 and 2.7, respectively, and are again compared with results obtained by Wu [12]. One observes in Fig. 2.6 that the agreement of the results is excellent if one ignores the (apparently) non-physical oscillations exhibited by the ϕ-component of current in Wu's solution. Note that the results obtained with DBR for $N = 30$ are presented as continuous curves, rather than discrete points. We comment that the data points nearest the points where the surface meets the body axis have been ignored and the curves have been extrapolated in this region. The data points which were ignored, however, have been plotted in the figure as dots which do not lie on the continuous curve. All other

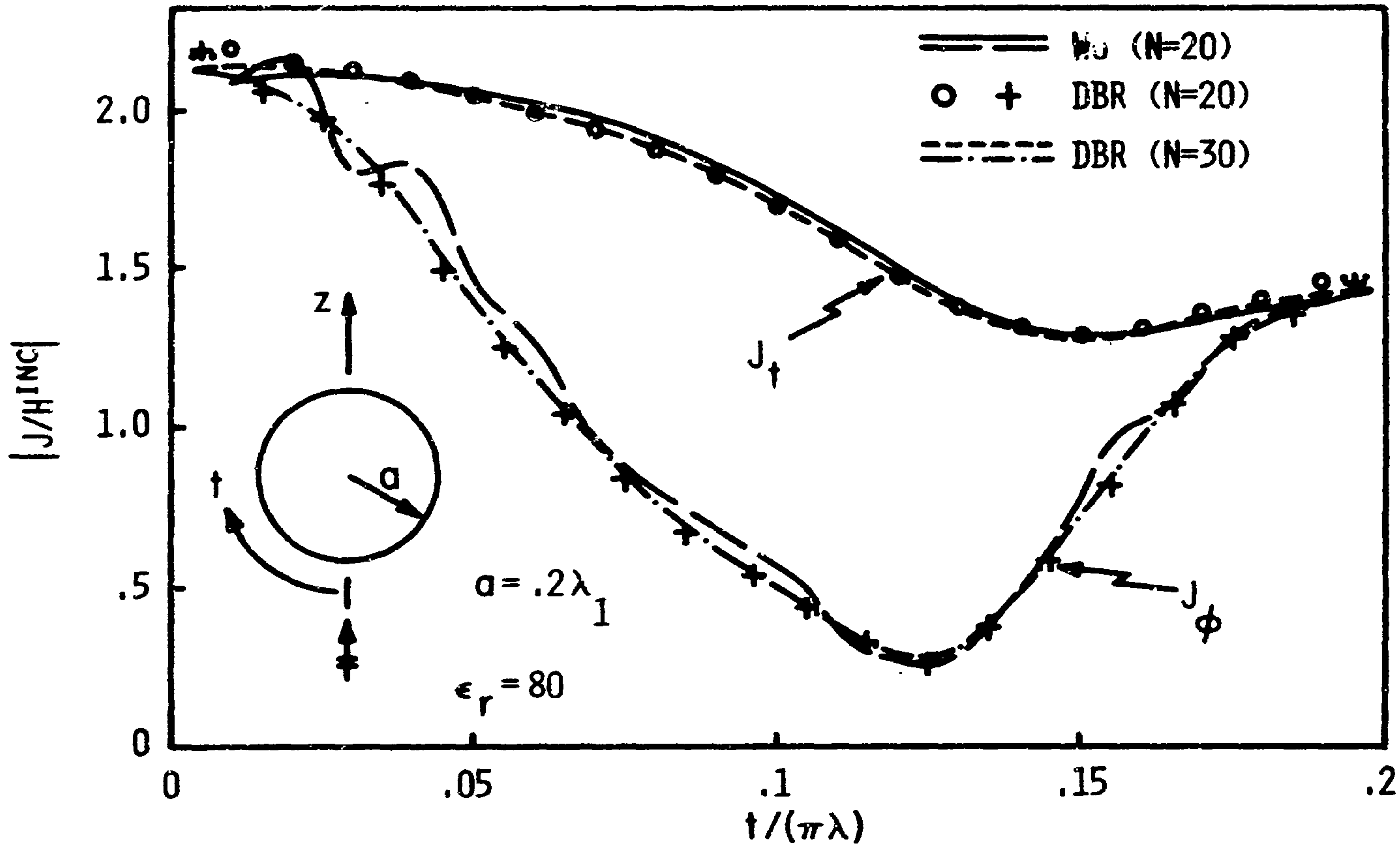

Figure 2.6. Electric surface current distribution on a dielectric sphere illuminated by an axially incident plane wave.

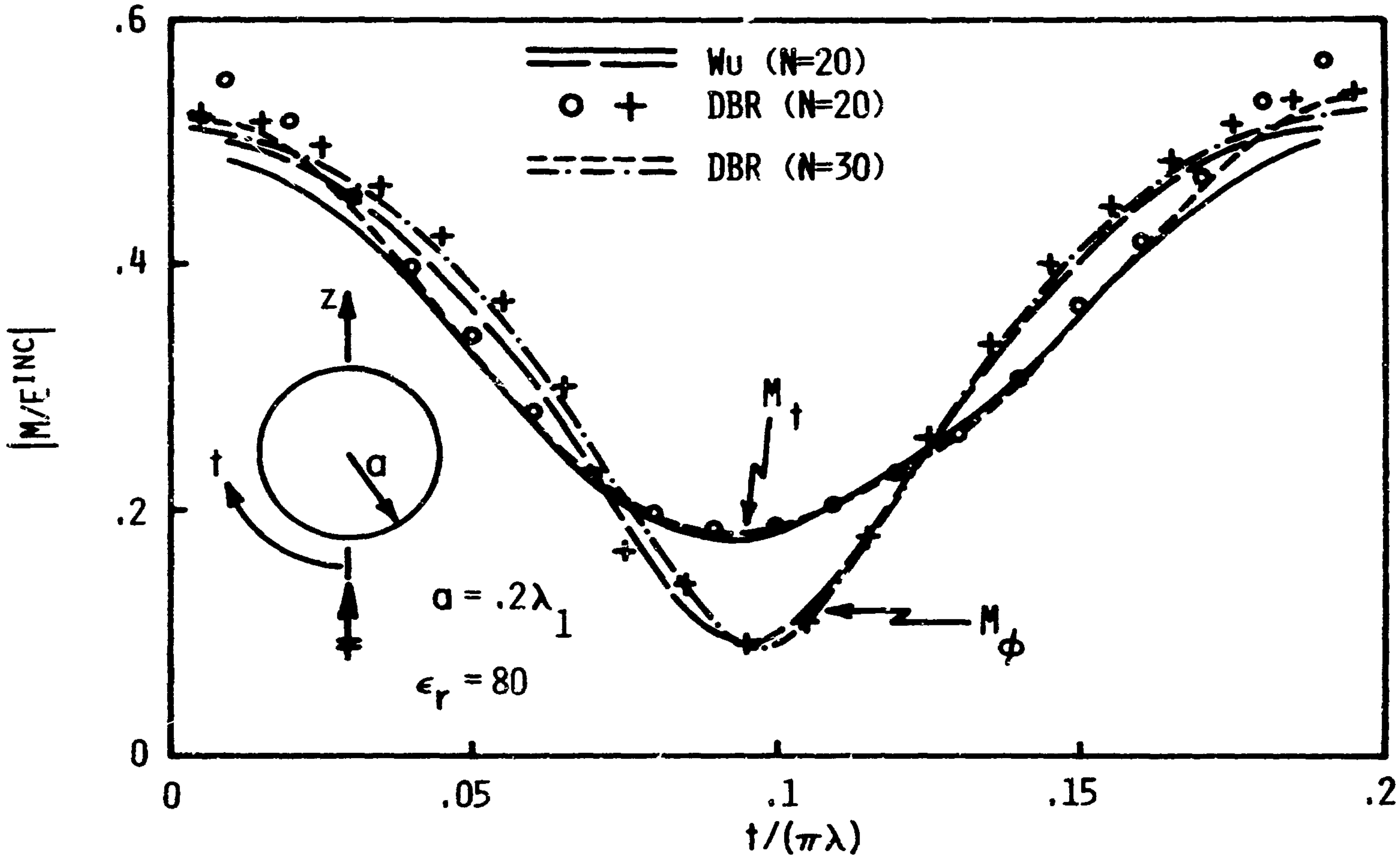

Figure 2.7. Magnetic surface current distribution on a dielectric sphere illuminated by an axially incident plane wave.

data points lie on the curves. The results for the magnetic current (Fig. 2.7) are also in good agreement. For the ϕ-component of magnetic current, however, the data obtained with DBR for $N = 20$ exhibit a slight downward bulge near $t/(\pi\lambda) \approx .075$, whereas Wu's results show no such behavior. The bulge disappears when more unknowns are used, as illustrated by the curve for $N = 30$.

In Fig. 2.8 numerical data are compared with the exact solution for the current densities on a "vacuum dielectric" ($\varepsilon_r = 1$) cylinder. The exact solution is, of course, given by

$$\bar{J} = \hat{n} \times \bar{H}^{inc}$$

$$\bar{M} = \bar{E}^{inc} \times \hat{n} \qquad .$$

The numerical data agree reasonably well with the exact solution when one considers the fact that the ϕ-components of current must exhibit a step-function jump in magnitude at the edges of the cylinder. The ability of a computer code to model such a case accurately may be important if one wishes to consider objects of very low dielectric contrast. Figs. 2.9 and 2.10 show calculated electric and magnetic current distributions, respectively, for the same size cylinder when the dielectric constant is increased to $\varepsilon_r = 4$.

We next consider numerical results for two perfectly

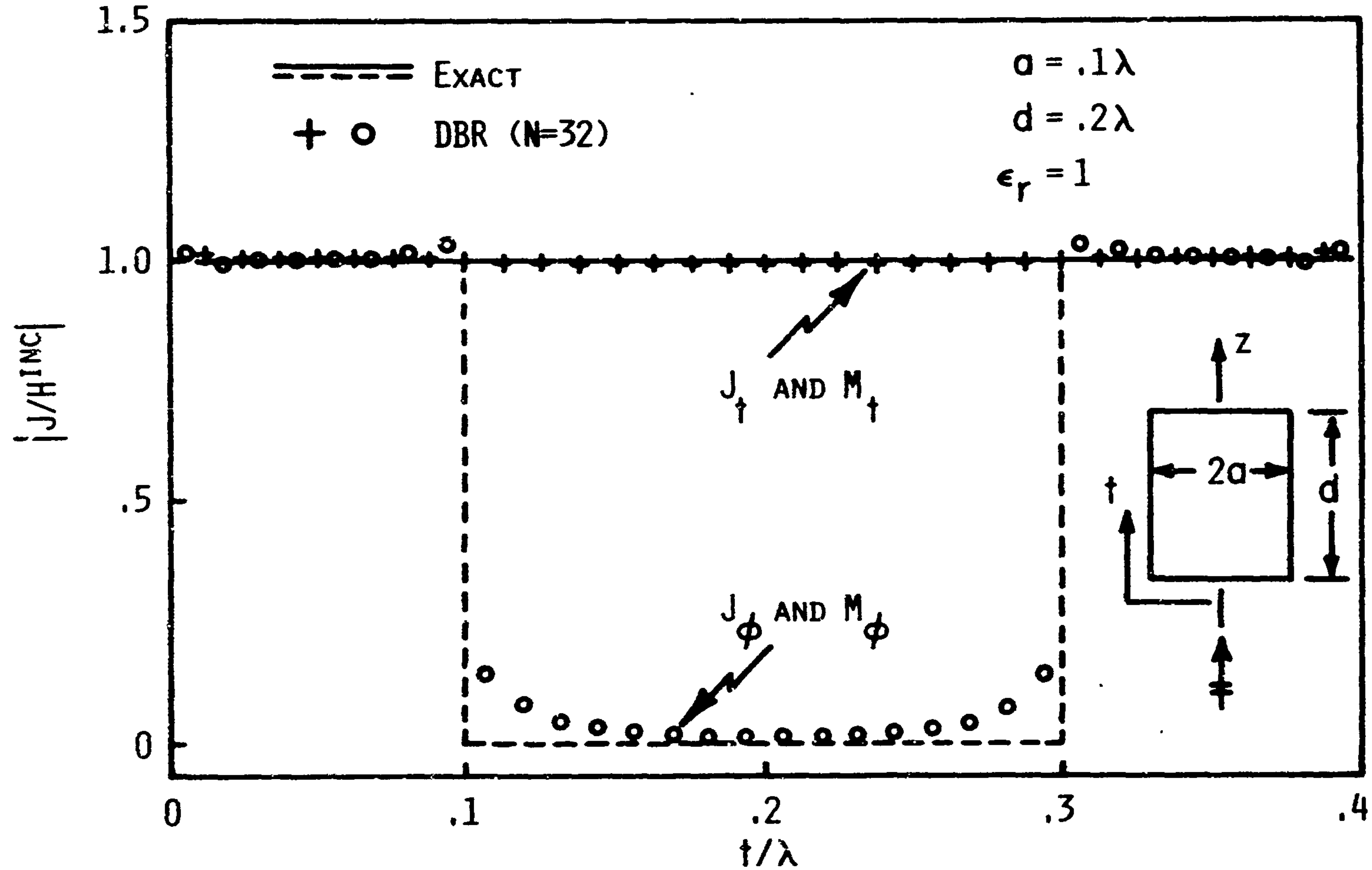

Figure 2.8. Electric and magnetic surface current distributions on a "vacuum dielectric" cylinder illuminated by an axially incident plane wave.

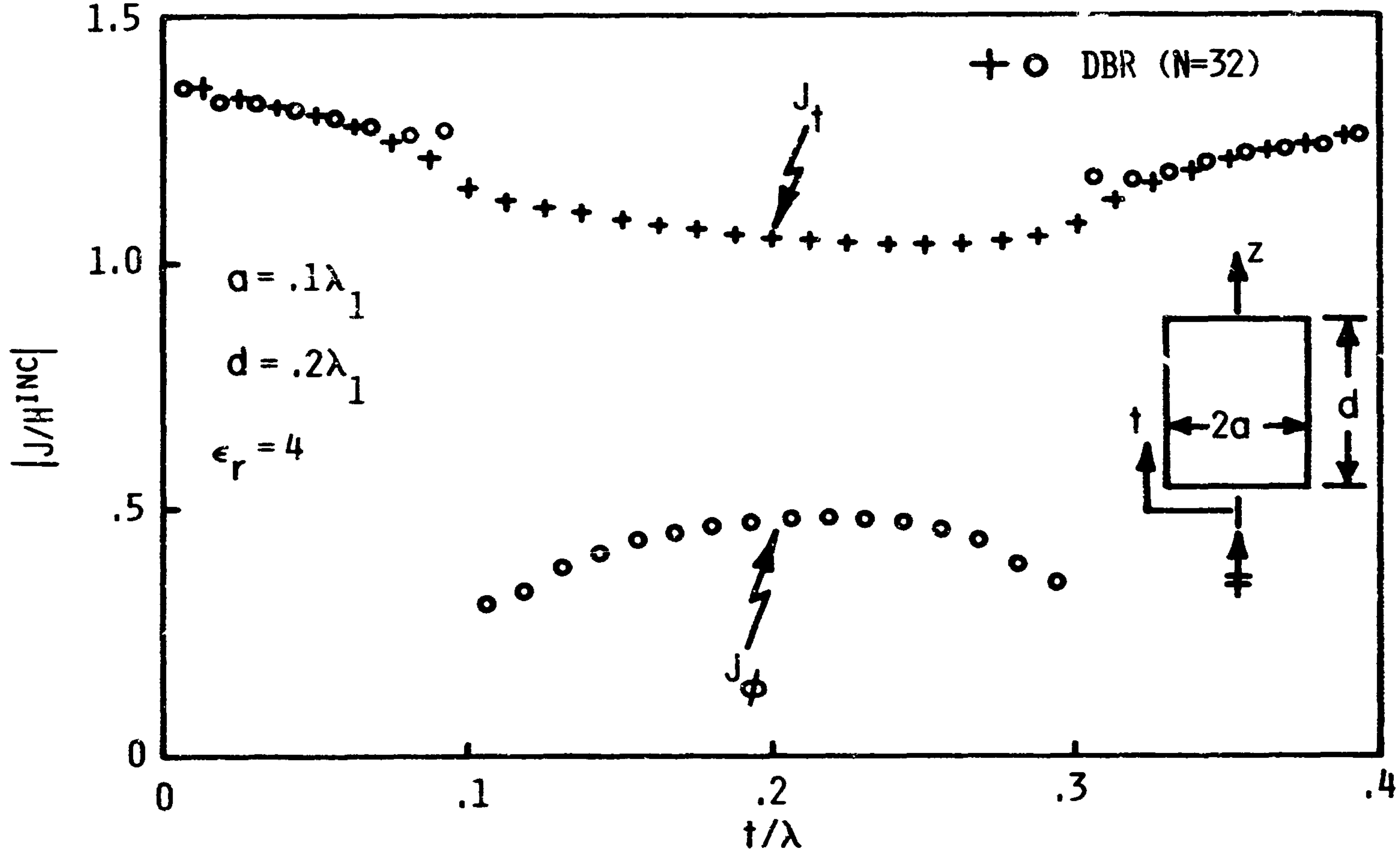

Figure 2.9. Electric surface current distribution on a dielectric cylinder illuminated by an axially incident plane wave.

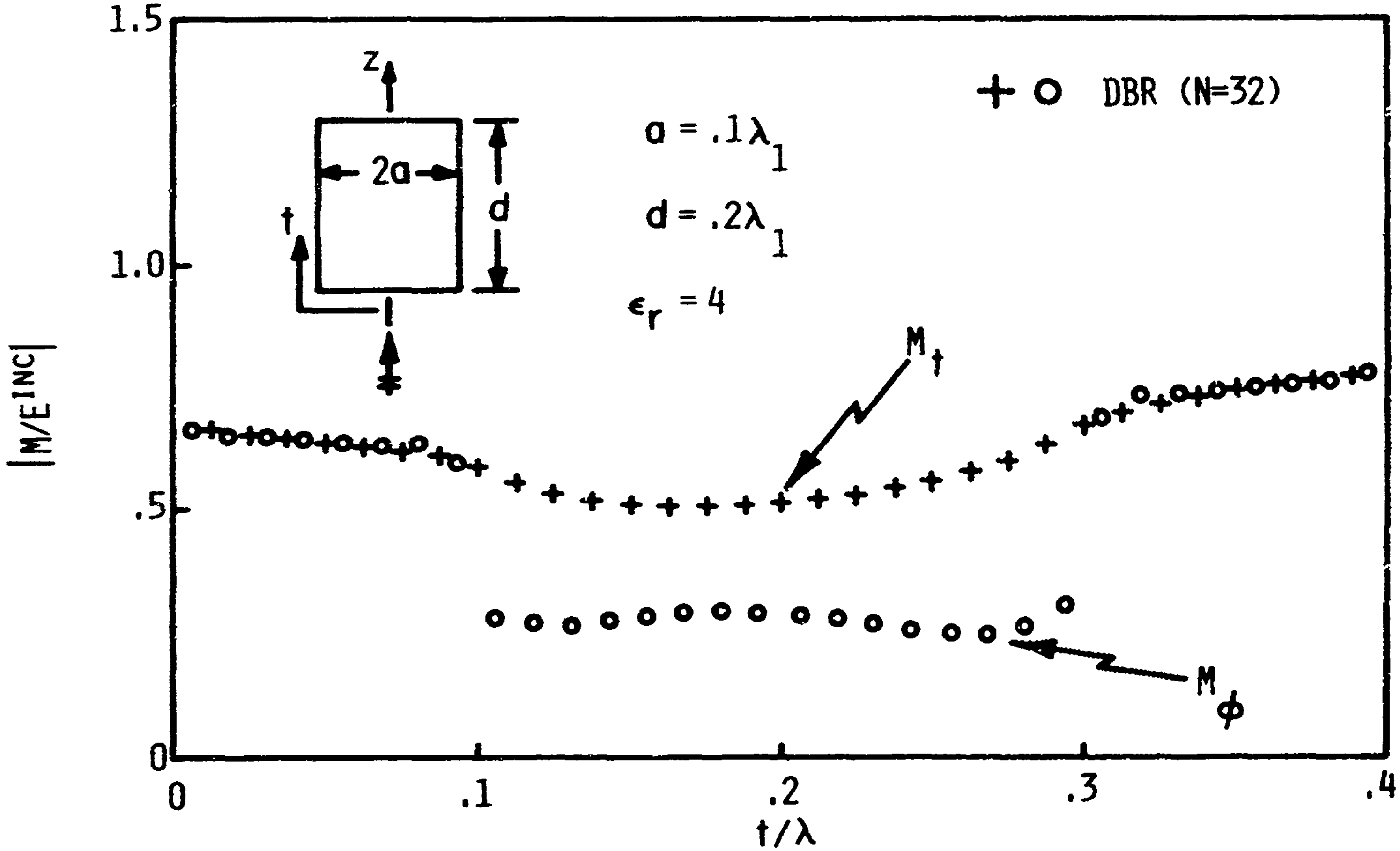

Figure 2.10. Magnetic surface current distribution on a dielectric cylinder illuminated by an axially incident plane wave.

conducting bodies. Fig. 2.11 illustrates the current distribution on a cone-sphere structure illuminated by an axially incident plane wave propagating from the cone tip toward the spherical cap. Numerical results obtained with DBR are compared with results of Mautz and Harrington [1] and Poggio and Miller [13]. The results of Mautz and Harrington are obtained via an EFIE formulation, while Poggio and Miller employed the MFIE. Results obtained with DBR are in excellent agreement with those obtained with the MFIE and do not exhibit the (apparently) non-physical oscillations in the current distributions obtained by Mautz and Harrington.

We also note at this point that there may exist a theoretical relationship between the t- and ϕ-components of current at the points where the surface meets the body axis. Such a relationship is of interest when treating body of revolution structures. Consider the region near the tip of the cone-sphere structure depicted in Fig. 2.11. The charge density near the tip for the m^{th} Fourier component is given by

$$\rho = \frac{1}{\omega r}\left[\frac{\partial}{\partial t}(rJ_t) + \frac{\partial}{\partial \phi}(J_\phi)\right]$$
$$= \frac{1}{\omega r}\left[J_t\frac{\partial}{\partial t}(r) + r\frac{\partial}{\partial t}(J_t) + \frac{\partial}{\partial \phi}(J_\phi)\right] ,$$

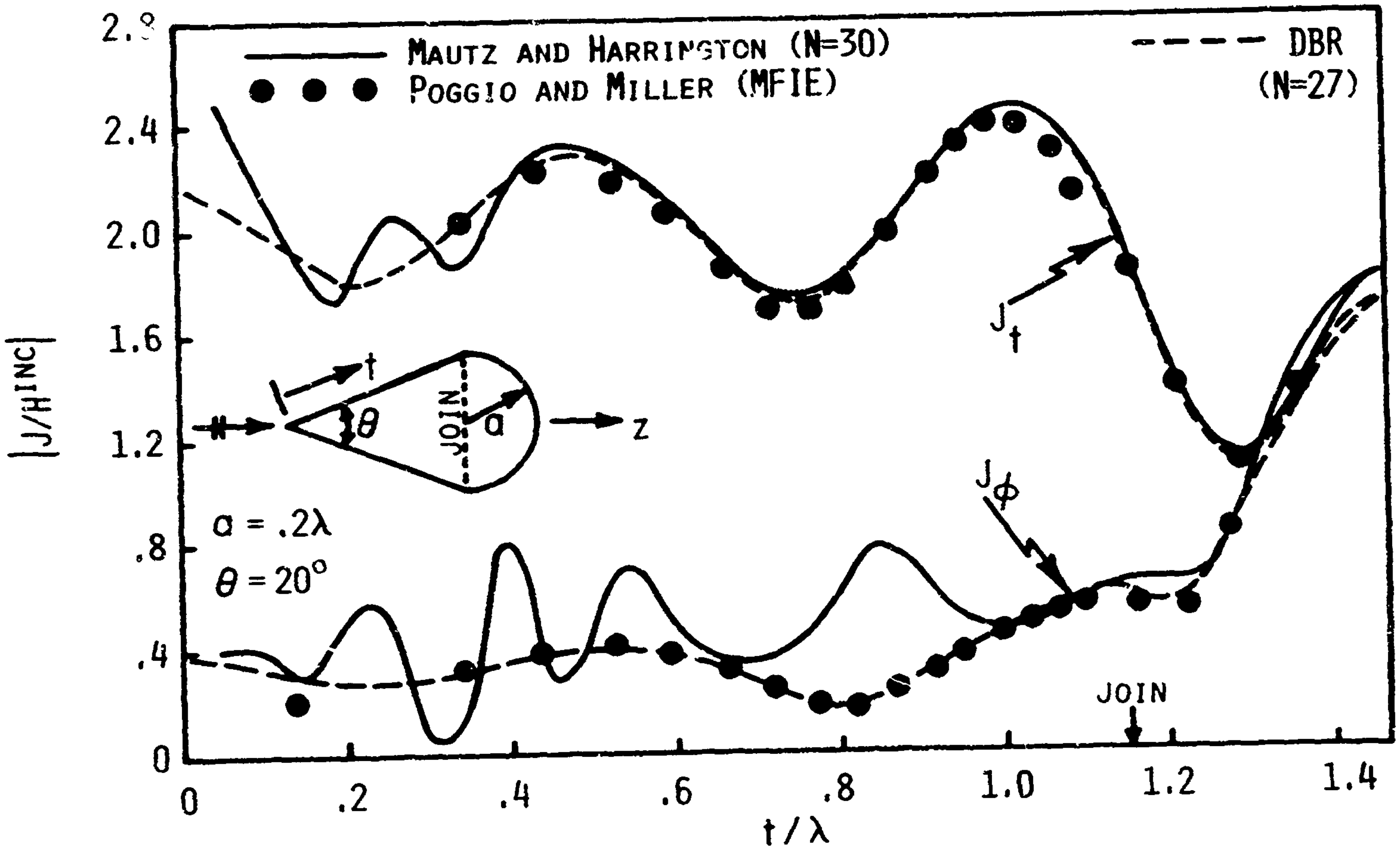

Figure 2.11. Surface current distribution on a perfectly conducting cone-sphere structure illuminated by a plane wave which is axially incident and propagates from the cone tip toward the spherical cap.

or

$$\rho = \frac{j}{\omega r}\left[J_t \sin(\theta/2) + r\frac{\partial}{\partial t}(J_t) + jmJ_\phi\right] \quad , \qquad (2.37)$$

where r is the cylindrical coordinate variable representing the distance from the z-axis and where, on the conical surface, $r = t\sin(\theta/2)$. The "total" charge is therefore

$$q = j\frac{2\pi}{\omega}\left[J_t \sin(\theta/2) + r\frac{\partial}{\partial t}(J_t) + jmJ_\phi\right] \quad . \qquad (2.38)$$

From symmetry considerations, however, it can be shown that the total charge at the points where the surface meets the body axis is zero for $m \neq 0$. At this point we rely on a visual inspection of the numerical results to infer that for $m = \pm 1$

$$\lim_{t \to 0} r\frac{\partial}{\partial t}(J_t) = 0 \quad . \qquad (2.39)$$

A rigorous theoretical investigation of the fields in the neighborhood of the cone tip is necessary to determine the validity of (2.39). Such an investigation, however, is seriously hampered by the lack of tabulated zeros of the associated Legendre function, and has therefore not been pursued in this work. With (2.38) we then have for $m = \pm 1$

$$J_t \sin(\theta/2) = \mp jJ_\phi \quad , \qquad (2.40)$$

which relates the value of the two current components at the point where the surface meets the body axis. The special case for $\theta = \pi$ and $m = \pm 1$ was developed previously by Mautz and Schuman [7]. Eq. (2.40) may be of some interest for "conically-tipped" surfaces when $m = \pm 1$. Referring to Fig. 2.11 again, one should note that results computed by means of DBR (which enforces no special conditions at the body axis) do indeed satisfy (within 5%) the condition (2.40) both at the cone tip and at the point on the spherical cap at the axis.

When the cone-sphere structure is illuminated by an axially incident plane wave propagating from the spherical cap to the cone tip, the current distributions shown in Fig. 2.12 are obtained. Results computed with DBR are again compared with those of Mautz and Harrington [1] and Poggio and Miller [13]. For the ϕ-component of current, agreement with the MFIE approach is again excellent, except very near the cone tip. However, for the t-component of current, the MFIE approach yields results which appear to oscillate slightly. Results obtained by means of DBR show essentially no oscillations. Also, the computed results again satisfy the condition (2.40) both at the cone tip and at the point on the spherical cap at the axis.

Fig. 2.13 illustrates the current distribution on a

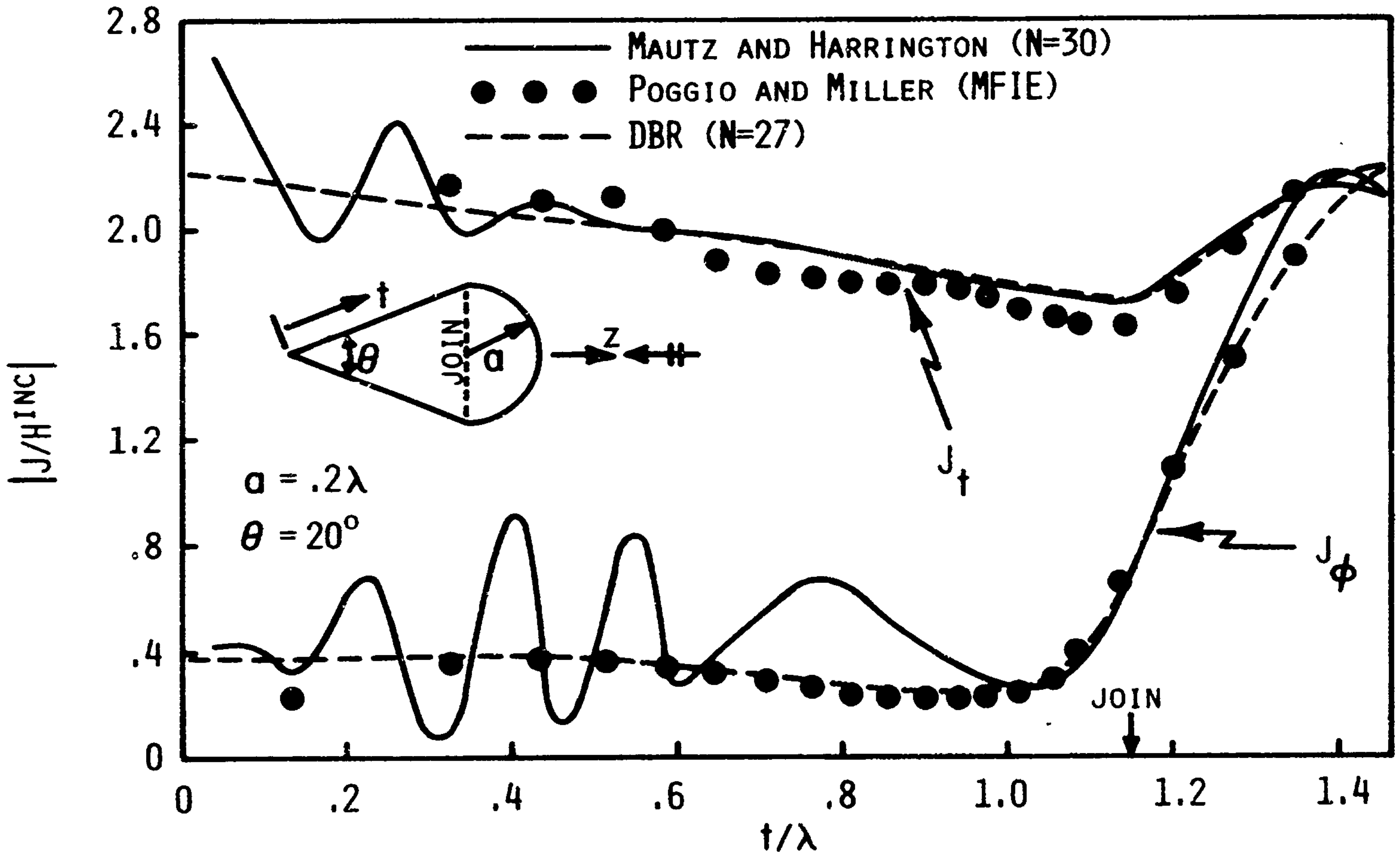

Figure 2.12. Surface current distribution on a perfectly conducting cone-sphere structure illuminated by a plane wave which is axially incident and propagates from the spherical cap toward the cone tip.

perfectly conducting open-ended cylinder subject to an axially incident plane wave. The results computed with DBR are compared with those of Davis, which were obtained using a set of hybrid integral equations [14]. Davis apparently encountered stability problems in the solution of the usual EFIE, and developed the hybrid equations as a means to avoid these problems. A comparison of the data in Fig. 2.13 should confirm that the formulation presented in this work apparently does not suffer from stability problems of the kind encountered by Davis. This problem also presents the opportunity to check the *a posteriori* correction for pulses which represent singular currents near edges [15]. The two square symbols near the edges of the plot in Fig. 2.13 represent computed currents which have been corrected *a posteriori* by dividing the computed current magnitude by $\sqrt{2}$. These corrected values are in excellent agreement with the results obtained by Davis, who used spline functions containing the edge condition. On the other hand, if one wishes to calculate the fields using the computed currents, it is probably better not to correct the value of the edge current, since the computed (uncorrected) value should accurately represent the current *moment* over the pulse region.

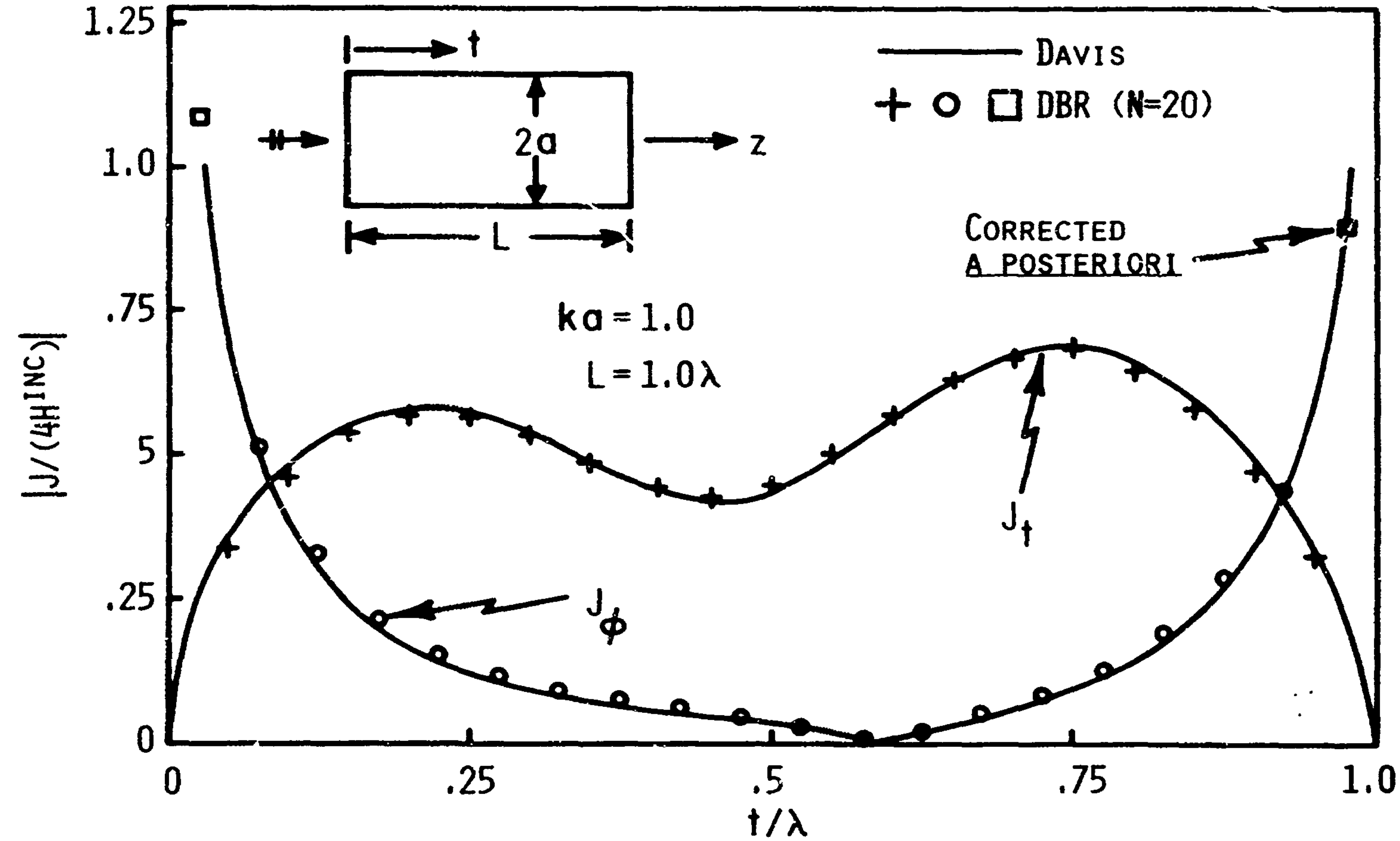

Figure 2.13. Surface current distribution on a perfectly conducting open-ended cylinder illuminated by an axially incident plane wave.

Section III

CONCLUSION

In this report we have presented a simple and efficient numerical formulation for determining the currents induced on both perfectly conducting and dielectric bodies of revolution. The formulation has been implemented in a computer code (DBR) which is described and listed in Appendix B. Numerical results have been presented for the current induced on several types of bodies of revolution and the excellent agreement of the results with other available data has been demonstrated. It has also been noted that other researchers have encountered stability problems with some body of revolution configurations. There has been no evidence of solution instability for any structural geometry when the procedures described in this work are employed. A slight "glitch" in the solution for the current has been detected at points where the surface meets the axis of the body of revolution. However, this glitch is relatively small and should have no significant effect on integral functionals of the current such as fields, radar cross-section, etc. Furthermore, it has been demonstrated that the techniques described in this work provide accurate treatment of sharp bends and/or knife-like

edges in the structure.

The numerical formulation presented in this report serves as a basis for a very sophisticated numerical model of the missile/plume structure in which the inhomogeneous plume is modeled by layers of homogeneous material. This sophisticated numerical model now apperars to have been successfully implemented and will be the subject of a forthcoming report.

APPENDIX A

SINGULARITY ANALYSES FOR SELF TERMS OF THE BODY OF REVOLUTION FORMULATION

When calculating elements of the impedance matrix for the body of revolution via (2.25), one should be aware that the integrands of the integral functions ψ and U may possess a singularity when the field point is within the source region. In this appendix each integrand is investigated to determine if a singularity is present and, if so, a numerical treatment of the integral function is presented. In all of the investigations we employ the following coordinate parameterization valid for $t_{j-1} \le t \le t_j$:

$$z = z_{j-1} + \ell \cos \gamma_j \qquad \text{(A.1a)}$$

$$\rho = \rho_{j-1} + \ell \sin \gamma_j \quad , \qquad \text{(A.1b)}$$

$$0 \le \ell \le \Delta t_j \ ,$$

where $\ell = t - t_{j-1}$. For all self terms we then have that

$$(z - z') = (\ell - \ell')\cos \gamma_j \qquad \text{(A.2a)}$$

$$(\rho - \rho') = (\ell - \ell')\sin \gamma_j \quad . \qquad \text{(A.2b)}$$

A.1 Singularity Analysis of the ψ and ψ^ρ Integral Functions

For a self term, the ψ integral function of (2.23a) may be written

$$\psi_i = \int_{\ell_1}^{\ell_2}\int_{-\pi}^{\pi} \frac{e^{-jk_iR_0}}{R_0}\cos(m\xi)\, d\xi d\ell' \quad , \qquad i = 1,2 \ , \tag{A.3a}$$

where

$$R_0 = [(\rho-\rho')^2 + 2\rho\rho'(1-\cos\xi) + (z-z')^2]^{\frac{1}{2}} . \tag{A.3b}$$

As $t \to t'$ and $\xi \to 0$, $R_0 \to 0$ and the integrand of (A.3a) is clearly singular. Thus we express ψ as

$$\psi_i = I_{1_i} + I_{2_i} \quad , \tag{A.4}$$

where

$$I_{1_i} = \int_{\ell_1}^{\ell_2}\int_{-\pi}^{\pi}\left[\frac{e^{-jk_iR_0}}{R_0}\cos(m\xi) - \frac{1}{R_0}\right] d\xi d\ell' \tag{A.5a}$$

$$I_{2_i} = \int_{\ell_1}^{\ell_2}\int_{-\pi}^{\pi} \frac{1}{R_0}\, d\xi d\ell' \quad , \tag{A.5b}$$

and where the integrand of I_{1_i} is no longer singular. Furthermore, we have that

$$R_0 = [(\rho - \rho')^2 + (z - z')^2 + 4\rho\rho'\sin^2(\xi/2)]^{\frac{1}{2}}$$
$$= R_1[1 + \beta_1^2\sin^2(\xi/2)]^{\frac{1}{2}} \quad ,$$

where

$$R_1 = [(\rho - \rho')^2 + (z - z')^2]^{\frac{1}{2}}$$
$$\beta_1 = \frac{2[\rho\rho']^{\frac{1}{2}}}{R_1} \quad .$$

Thus, with a simple change of variables, I_{2_i} may be written as [16]

$$I_{2_i} = 4\int_{\ell_1}^{\ell_2}\int_0^{\frac{\pi}{2}} \frac{1}{R_1[1 + \beta_1^2\sin^2\zeta]^{\frac{1}{2}}}\, d\zeta d\ell'$$
$$= 4\int_{\ell_1}^{\ell_2} \frac{1}{R_1[1 + \beta_1^2]^{\frac{1}{2}}}\, K\left(\frac{1}{[1 + \beta_1^2]^{\frac{1}{2}}}\right) d\ell'$$
$$= 4\int_{\ell_1}^{\ell_2} \frac{1}{R_2}\, K(\beta_2)\, d\ell' \quad , \qquad \text{(A.6)}$$

where $K(\beta_2)$ is the complete elliptic integral of the first kind defined by

$$K(u) = \int_0^{\frac{\pi}{2}} \frac{1}{[1 - u^2 \sin^2\phi]^{\frac{1}{2}}} \, d\phi \quad , \tag{A.7a}$$

and

$$R_2 = [(\rho + \rho')^2 + (z - z')^2]^{\frac{1}{2}} \tag{A.7b}$$

$$\beta_2^* = \frac{2[\rho\rho']^{\frac{1}{2}}}{R_2} \quad . \tag{A.7c}$$

The integrand of (A.6) is still singular, however, and near the singularity at $t = t'$ varies as

$$\frac{1}{R_2} K(\beta_2) \xrightarrow[t \to t']{} \frac{1}{2\rho} [\ln 4 + \ln(R_2) - \ln(R_1)] \; . \tag{A.8}$$

Only the last term is singular, so we add and subtract the singular term from (A.6) to obtain

$$I_{2_i} = I_{2_i}^a + I_{2_i}^b \quad , \tag{A.9}$$

where

$$I_{2_i}^a = 4\int_{\ell_1}^{\ell_2} \left[\frac{1}{R_2} K(\beta_2) + \frac{1}{2\rho} \ln(R_1)\right] d\ell' \tag{A.10a}$$

$$I_{2_i}^b = -\frac{2}{\rho}\int_{\ell_1}^{\ell_2} \ell n(R_1)\, d\ell' \quad . \tag{A.10b}$$

The integral $I_{2_i}^a$ no longer has a singular integrand and the integral $I_{2_i}^b$ can be evaluated analytically by using the parameterization (A.1) as follows:

$$I_{2_i}^b = -\frac{2}{\rho}\int_{\ell_1}^{\ell_2} \ell n(R_1)\, d\ell'$$

$$= -\frac{2}{\rho}\int_{\ell_1}^{\ell_2} \ell n|\ell - \ell'|\, d\ell'$$

$$= \frac{2}{\rho}[(\ell_2 - \ell_1) - (\ell_2 - \ell)\ell n(\ell_2 - \ell)$$

$$- (\ell - \ell_1)\ell n(\ell - \ell_1)] \quad . \tag{A.11}$$

The integrals I_{1_i} and $I_{2_i}^a$ can thus be integrated numerically, while $I_{2_i}^b$ is given by (A.11), and we have

$$\psi_i = I_{1_i} + I_{2_i}^a + I_{2_i}^b \quad . \tag{A.12}$$

We comment here that for *non-self* terms it is more

convenient to calculate ψ_i as

$$\psi_i = I_{1_i} + I_{2_i} \qquad , \tag{A.13}$$

where I_{2_i} is defined by (A.6), than to compute ψ_i directly from (A.3). Following the procedures described above it is trivial to show that the self term of ψ_i^ρ is calculated via

$$\psi_i^\rho = I_{1_i}^\rho + \rho I_{2_i}^a + \rho I_{2_i}^b \qquad , \tag{A.14}$$

where

$$I_{1_i}^\rho = \int_{\ell_1}^{\ell_2} \int_{-\pi}^{\pi} \left\{ \frac{e^{-jk_1R_0}}{R_0} \rho' \cos(m\xi) - \frac{\rho}{R_0} \right\} d\xi d\ell' \quad . \tag{A.15}$$

A.2 Singularity Analysis of the Integral Function U_0

The definition of the integral function U_0 is given by (2.26a) as

$$U_0 = \int_{\ell_1}^{\ell_2} \int_{-\pi}^{\pi} - \frac{\sin(m\xi)\sin\xi}{R_0^3} \Big[(1 + jk_1R_0)\, e^{-jk_1R_0} + (1 + jk_2R_0)\, e^{-jk_2R_0} \Big] d\xi d\ell' \qquad , \tag{A.16}$$

where R_0 is defined by (A.3b). As stated in Section II, all of the integral functions U may be Cauchy Principal Value integrals. Thus one may evaluate the integrals by direct numerical integration, if the quadrature points are chosen in accordance with the definition of the Cauchy Principal Value Integral. On the other hand, evaluation of the integrals in this manner may result in the subtraction of very large numerical values of the integrands and hence lead to possible loss of precision in the calculations. To avoid such problems we analyze the U integral functions in the manner presented in Section A.1. Near the singular point the integrand of (A.16) can be shown to vary as

$$\frac{-2m\xi^2}{\left[(\ell - \ell')^2 + \rho^2\xi^2\right]^{(3/2)}} \quad .$$

Therefore we write

$$U_0 = I_3 + I_4 \quad , \tag{A.17}$$

where

$$I_3 = \int_{\ell_1}^{\ell_2} \int_{-\pi}^{\pi} \left\{ -\frac{\sin(m\xi)\sin\xi}{R_0^3} \left[(1 + jk_1R_0)\ e^{-jk_1R_0} \right.\right.$$

$$\left.\left. + (1 + jk_2R_0)\ e^{-jk_2R_0} \right] - \frac{-2m\xi^2}{\left[(\ell - \ell')^2 + \rho^2\xi^2 \right]^{(3/2)}} \right\} d\xi d\ell' \tag{A.18a}$$

$$I_4 = \int_{\ell_1}^{\ell_2} \int_{-\pi}^{\pi} \frac{-2m\xi^2}{\left[(\ell - \ell')^2 + \rho^2\xi^2 \right]^{(3/2)}}\ d\xi d\ell' \quad . \tag{A.18b}$$

The integrand of I_3 is non-singular and can be evaluated numerically. The integral I_4 can be integrated analytically and is given by

$$I_4 = -\frac{4m}{\rho^3} \left\{ (\ell - \ell_1)\ell n \left[\rho\pi + \sqrt{(\ell - \ell_1)^2 + \rho^2\pi^2} \right] \right.$$

$$+ (\ell_2 - \ell)\ell n \left[\rho\pi + \sqrt{(\ell_2 - \ell)^2 + \rho^2\pi^2} \right]$$

$$\left. - (\ell - \ell_1)\ell n(\ell - \ell_1) - (\ell_2 - \ell)\ell n(\ell_2 - \ell) \right\} \quad . \tag{A.19}$$

Eq. (A.17) then provides the desired result.

A.3 Singularity Analysis of the Integral Functions U_1 and U_1^ρ

The integrand of the integral function U_1, as defined by (2.26b), is identical with the integrand of the integral function U_0 except for the product factor $(z - z')$. It should therefore be clear from the analysis of U_0 that as $R_0 \to 0$ the integrands of U_1 and U_1^ρ are proportional to

$$\frac{-2m\xi^2(\ell - \ell')\cos\gamma'}{\left[(\ell - \ell')^2 + \rho^2\xi^2\right]^{(3/2)}} ,$$

which is non-singular and hence can be integrated numerically.

A.4 Singularity Analysis of the Integral Functions U_2 and U_2^ρ

By comparison with the integrand of U_1, one can immediately see that as $R_0 \to 0$ the integrand of U_2 approaches

$$\frac{-2(\ell - \ell')\cos\gamma'}{\left[(\ell - \ell')^2 + \rho^2\xi^2\right]^{(3/2)}} .$$

This function is singular and we therefore attempt to compute U_2 as

$$U_2 = I_5 + I_6 \quad , \tag{A.20}$$

where

$$I_5 = \int_{\ell_1}^{\ell_2} \int_{-\pi}^{\pi} \left\{ -\frac{(z - z')\cos(m\xi)\cos\xi}{R_0^3} \left[(1 + jk_1R_0)\, e^{-jk_1R_0} \right.\right.$$

$$\left.\left. + (1 + jk_2R_0)\, e^{-jk_2R_0} \right] + \frac{2(\ell - \ell')\cos\gamma'}{\left[(\ell - \ell')^2 + \rho^2\xi^2\right]^{(3/2)}} \right\} d\xi d\ell' \tag{A.21a}$$

$$I_6 = \int_{\ell_1}^{\ell_2} \int_{-\pi}^{\pi} -\frac{2(\ell - \ell')\cos\gamma'}{\left[(\ell - \ell')^2 + \rho^2\xi^2\right]^{(3/2)}} \, d\xi d\ell' \quad . \tag{A.21b}$$

The integrand of I_5 is non-singular and may be integrated numerically. Analytical evaluation of I_6 yields

$$I_6 = \frac{4\cos\gamma'}{\rho} \left\{ \ell n \left[\frac{\rho\pi + \sqrt{(\ell - \ell_1)^2 + \rho^2\pi^2}}{\rho\pi + \sqrt{(\ell - \ell_2)^2 + \rho^2\pi^2}} \right] + \ell n \left| \frac{\ell - \ell_2}{\ell - \ell_1} \right| \right\} , \tag{A.22}$$

which is infinite if $\ell = \ell_1$ or $\ell = \ell_2$. These values may occur in the evaluation of (2.25g), so U_2 has a non-integrable singularity. The consequences of this result are discussed in Section A.9. We similarly express

$$U_2^\rho = I_7 + \rho I_6 \quad , \tag{A.23}$$

where

$$I_7 = \int_{\ell_1}^{\ell_2} \int_{-\pi}^{\pi} \left\{ - \frac{(z - z')\rho' \cos(m\xi)\cos\xi}{R_0^3} \left[(1 + jk_1 R_0)\, e^{-jk_1 R_0} + (1 + jk_2 R_0)\, e^{-jk_2 R_0} \right] + \frac{2\rho(\ell - \ell')\cos\gamma'}{\left[(\ell - \ell')^2 + \rho^2 \xi^2\right]^{(3/2)}} \right\} d\xi\, d\ell' \quad . \tag{A.24}$$

Thus U_2^ρ also has a non-integrable singularity and will be treated in Section A.9.

A.5 Singularity Analysis of the Integral Function U_3^ρ

By direct comparison with U_0 one can immediately write

$$U_3^\rho = I_8 + \rho I_4 \quad , \tag{A.25}$$

where

$$I_8 = \int_{\ell_1}^{\ell_2} \int_{-\pi}^{\pi} \left\{ - \frac{\rho' \sin(m\xi)\sin\xi}{R_0^3} \left[(1 + jk_1 R_0)\, e^{-jk_1 R_0} + (1 + jk_2 R_0)\, e^{-jk_2 R_0} \right] + \frac{2m\rho\xi^2}{\left[(\ell - \ell')^2 + \rho^2 \xi^2\right]^{(3/2)}} \right\} d\xi\, d\ell' \quad , \tag{A.26}$$

and where I_4 is given by (A.19). The integral I_8 has a non-singular integrand and can be integrated numerically.

A.6 Singularity Analysis of the Integral Function U_4^ρ

By analogy with U_2^ρ one can compute

$$U_4^\rho = I_9 + I_{10} \quad , \tag{A.27}$$

where

$$I_9 = \int_{\ell_1}^{\ell_2}\int_{-\pi}^{\pi}\left\{-\frac{\rho'(\rho-\rho')\cos(m\xi)}{R_0^3}\left[(1+jk_1R_0)\ e^{-jk_1R_0} + (1+jk_2R_0)\ e^{-jk_2R_0}\right] + \frac{2\rho(\ell-\ell')\sin\gamma'}{\left[(\ell-\ell')^2+\rho^2\xi^2\right]^{(3/2)}}\right\} d\xi d\ell' \tag{A.28}$$

$$I_{10} = 4\sin\gamma'\left\{\ell n\left[\frac{\rho\pi+\sqrt{(\ell-\ell_1)^2+\rho^2\pi^2}}{\rho\pi+\sqrt{(\ell-\ell_2)^2+\rho^2\pi^2}}\right] + \ell n\left|\frac{\ell-\ell_2}{\ell-\ell_1}\right|\right\}. \tag{A.29}$$

The integral I_9 can be computed numerically, but I_{10} is infinite if $\ell = \ell_1$ or $\ell = \ell_2$, and is therefore considered in Section A.9.

A.7 Singularity Analysis of the Integral Function U_5 and U_5^ρ

The integral function U_5 is defined by (2.26h) as

$$U_5 = \int_{\ell_1}^{\ell_2} \int_{-\pi}^{\pi} - \frac{\cos(m\xi)\sin^2(\xi/2)}{R_0^3} \left[(1 + jk_1R_0)\, e^{-jk_1R_0} + (1 + jk_2R_0)\, e^{-jk_2R_0} \right] d\xi d\ell' \quad . \qquad \text{(A.30)}$$

As $R_0 \to 0$ the integrand of U_5 approaches

$$\frac{-\xi^2}{2\left[(\ell - \ell')^2 + \rho^2\xi^2\right]^{(3/2)}} \quad .$$

The integrand is therefore singular and we compute U_5 as

$$U_5 = I_{11} + I_{12} \quad , \qquad \text{(A.31)}$$

where

$$I_{11} = \int_{\ell_1}^{\ell_2} \int_{-\pi}^{\pi} \left\{ - \frac{\cos(m\xi)\sin^2(\xi/2)}{R_0^3} \left[(1 + jk_1R_0)\, e^{-jk_1R_0} + (1 + jk_2R_0)\, e^{-jk_2R_0} \right] + \frac{\xi^2}{2\left[(\ell - \ell')^2 + \rho^2\xi^2\right]^{(3/2)}} \right\} d\xi d\ell' \qquad \text{(A.32a)}$$

$$I_{12} = \int_{\ell_1}^{\ell_2} \int_{-\pi}^{\pi} - \frac{\xi^2}{2\left[(\ell - \ell')^2 + \rho^2 \xi^2\right]^{(3/2)}} \, d\xi d\ell'$$

$$= -\frac{1}{\rho^3} \left\{ (\ell - \ell_1) \ell n \left[\rho\pi + \sqrt{(\ell - \ell_1)^2 + \rho^2 \pi^2} \right] \right.$$

$$+ (\ell_2 - \ell) \ell n \left[\rho\pi + \sqrt{(\ell_2 - \ell)^2 + \rho^2 \pi^2} \right]$$

$$\left. - (\ell - \ell_1) \ell n (\ell - \ell_1) - (\ell_2 - \ell) \ell n (\ell_2 - \ell) \right\} . \quad \text{(A.32b)}$$

The integral I_{11} is integrated numerically since the integrand is now non-singular. Similarly,

$$U_5^\rho = I_{13} + \rho I_{12} \quad , \quad \text{(A.33)}$$

where

$$I_{13} = \int_{\ell_1}^{\ell_2} \int_{-\pi}^{\pi} \left\{ - \frac{\rho' \cos(m\xi) \sin^2(\xi/2)}{R_0^3} \left[(1 + jk_1 R_0) \, e^{-jk_1 R_0} \right. \right.$$

$$\left. \left. + (1 + jk_2 R_0) \, e^{-jk_2 R_0} \right] + \frac{\rho \xi^2}{2\left[(\ell - \ell')^2 + \rho^2 \xi^2\right]^{(3/2)}} \right\} d\xi d\ell' .$$

(A.34)

A.8 Singularity Analysis of the Integral Function U_6

Analysis of the integral function U_6 is directly analogous to that of U_4^ρ. We have

$$U_6 = I_{14} + \frac{1}{\rho} I_{10} \quad , \tag{A.35}$$

where

$$I_{14} = \int_{\ell_1}^{\ell_2} \int_{-\pi}^{\pi} \left\{ - \frac{(\rho - \rho')\cos(m\xi)\cos\xi}{R_0^3} \left[(1 + jk_1R_0)\, e^{-jk_1R_0} + (1 + jk_2R_0)\, e^{-jk_2R_0} \right] + \frac{2(\ell - \ell')\sin\gamma'}{\left[(\ell - \ell')^2 + \rho^2\xi^2 \right]^{(3/2)}} \right\} d\xi\, d\ell' \ , \tag{A.36}$$

and where I_{10} is given by (A.29). The integral I_{14} can be evaluated numerically, whereas the analytical result for I_{10} may be infinite and is therefore treated in the next section.

A.9 Elimination of Non-Integrable Singularities

The non-integrable singularities which have appeared in the analyses of the integral functions U_2, U_2^ρ, U_4^ρ, and U_6 are, of course, also non-physical. We therefore proceed

to determine how these apparent singularities may be removed. Consider now the quantity K which is the sum of the third and fifth terms of (2.25g). Then for a self term we have

$$\frac{4\pi K}{\Delta t_n} = -U_6 \cos\gamma_n + U_2 \sin\gamma_n$$

$$\doteq -I_{14}\cos\gamma_n - I_{10}\frac{\cos\gamma_n}{\rho} + I_5 \sin\gamma_n + I_6 \sin\gamma_n \quad . \tag{A.37}$$

Then note that

$$-I_{10}\frac{\cos\gamma_n}{\rho} + I_6 \sin\gamma_n = \left\{\frac{-4\cos\gamma_n \sin\gamma_n}{\rho} + \frac{4\sin\gamma_n \cos\gamma_n}{\rho}\right\} A \quad ,$$

$$\equiv 0 \tag{A.38a}$$

where

$$A = \ell n \left[\frac{\rho\pi + \sqrt{(\ell - \ell_1)^2 + \rho^2\pi^2}}{\rho\pi + \sqrt{(\ell - \ell_2)^2 + \rho^2\pi^2}}\right] + \ell n \left|\frac{\ell - \ell_2}{\ell - \ell_1}\right| \quad . \tag{A.38b}$$

Thus, the result for K does converge as expected. The numerical procedure for the evaluation of the integral functions U_2 and U_6 is therefore to compute

$$U_2 = I_5 \tag{A.39a}$$

$$U_6 = I_{14} \quad , \tag{A.38b}$$

thus removing the canceling non-integrable terms from the evaluation of the remaining terms in the integral.

If we next consider the quantity K which is the sum of the first and third terms of (2.25f), then for a self term we have, for example,

$$2K = \chi_c(\Delta t_n,\gamma_n)U_4^\rho - \chi_s(\Delta t_n,\gamma_n)U_2^\rho$$

$$= \chi_c(\Delta t_n,\gamma_n)(I_9 + I_{10}) - \chi_s(\Delta t_n,\gamma_n)(I_7 + \rho I_6) \ , \tag{A.40}$$

and

$$\chi_c(\Delta t_n,\gamma_n)I_{10} - \chi_s(\Delta t_n,\gamma_n)\rho I_6$$

$$= \{2[\Delta t_{n+1}\cos\gamma_{n+1} + \Delta t_n\cos\gamma_n]\sin\gamma_m$$

$$- 2[\Delta t_{n+1}\sin\gamma_{n+1} + \Delta t_n\sin\gamma_n]\cos\gamma_m\}A \ , \tag{A.41}$$

where A is defined by (A.38b) and where, for a self term, either $m = n$ or $m = n+1$. By way of example we choose $m = n$, which gives

$$\chi_c I_{10} - \chi_s\rho I_6 = 2A\Delta t_{n+1}\{\cos\gamma_{n+1}\sin\gamma_n - \sin\gamma_{n+1}\cos\gamma_n\} \ . \tag{A.42}$$

The trigonometric functions in (A.42) do not cancel unless $\gamma_n = \gamma_{n+1}$ (which implies that there is no bend in the surface

at $t = t_n$) and the result may therefore be infinite. The difficulty lies in the mathematical representation of the problem — the field point is placed between two current subdomains each of which has an independent associated current coefficient and this point may be at a bend in the surface where $\gamma_n \neq \gamma_{n+1}$. It is relatively easy to demonstrate that if the actual current on the structure appeared in the integrals, the integrals would be convergent if evaluated in the Cauchy Principal Value sense. To eliminate this problem, we define in this situation

$$U_2^{\rho} = I_7 \tag{A.43a}$$

$$U_4^{\rho} = I_9 \quad , \tag{A.43b}$$

thus removing the singular terms from the evaluation of the integrals. While this procedure is perhaps not entirely rigorous, it has been found to work satisfactorily.

Appendix B

DESCRIPTION AND LISTING OF THE CODE

The computer code DBR is designed to calculate the induced currents on a general body of revolution, which may be either a perfectly conducting or a dielectric body. Perfectly conducting bodies may be either "closed" or "open," i.e. the body surface may or may not intersect the axis of the body of revolution, respectively. Dielectric bodies must be closed. The code is not capable of treating bodies in which the generating arc forms a closed loop (such as for toroidal bodies) nor is it capable of treating bodies having a finite conductivity. The modifications necessary to include such cases, however, are fairly straightforward. The excitation which induces current on the body is assumed to be a plane wave for perfect conductors or dielectrics and/or a single delta-gap voltage source for perfect conductors. Any delta-gap voltage source is assumed to be ϕ-independent.

The output of the code for each excitation consists of a description of the input parameters and pertinent calculated parameters, listings of each Fourier component of current, and listings of sums over the computed Fourier current components observed in specified planes of constant ϕ.

The code itself contains sufficient information in comment cards for user operation. The general purpose auxiliary routines needed by the code are also listed in this appendix for the reader's convenience.

B.1 Program Operation

The program MAIN reads all input data except for input data which describes the generating arc. The input data is adequately described by comment cards in the program. The subroutine GENCUR is called to read the data describing the generating arc in terms of the ρ- and z-coordinates of the points t_n (see Fig. 2.2). GENCUR also calculates and stores vectors corresponding to the quantities $t_{n+\frac{1}{2}}$, Δt_n, and γ_n. The number of points used should be sufficient to adequately approximate the generating arc as well as to represent the variation of the current on the body. The body surface should not intersect the axis of the body of revolution except at the end points of the generating arc. If the surface does intersect the axis at either or both ends of the generating arc, then the points t_0 and/or t_{N+1} should be placed on the axis. The general form of the input data for a single case is given on the next page in terms of the program variables:

```
NFLDS          MODEB     MODEE        NGQ        IDB
LAMBDA         EPSR      MUR
THETA(1)       PHI(1)    ETHETA(1)    EPHI(1)    ANTFD(1)
    .
    .
    .
THETA(NFLDS)    PHI(NFLDS)   ETHETA(NFLDS) EPHI(NFLDS) ANTFD(NFLDS)
RHO(1)   Z(1)
RHO(2)   Z(2)
    .
    .
    .
RHO(NPTS)   Z(NPTS)
9999.0      9999.0
```

Note that the number of excitations entered must correspond to the input parameter NFLDS. On the other hand, the points read which describe the generating arc are terminated by the presence of the numbers 9999.0 and the quantity NPTS is calculated internally. Multiple cases may be run by placing a new data set immediately following the first data set.

Once all data is read for a particular case MAIN computes the dimensions required to run the case. If the absolute dimensions provided in MAIN are inadequate,execution is aborted for the case and data is read for a new case, if present; otherwise, SLTN is called to compute and print the results. The impedance matrix and drive vector are computed for each Fourier component by calls to the subroutines ZMATRX and CVFILL, respectively. The elements computed by these routines are twice those indicated by Eq. (2.25) and (2.30). The impedance matrix is inverted by the routine CSMINV and the current coefficients indicated in (2.17) are computed as

$I = Z^{-1}V$ for each Fourier component by the multiplication routine ICRMUL. The coefficients of the exponential series (2.17) are transformed to coefficients of a trigonometric series by the subroutine SUMOUT. The current coefficients are then printed and the process is repeated for the next Fourier component.

As mentioned previously, the elements of the impedance matrix calculated by the program are twice that indicated by (2.25). Except for this, the elements are calculated using the expressions (2.25) and the duality relations (2.14), where appropriate. The integral functions ψ and U appearing in (2.25) are calculated by numerical integration. Gaussian quadrature integration (subroutines CGQ1, CGQ1T) is used in the t-direction and the order of the approximation is controlled by the input parameter NGQ. Second order Gaussian quadrature (NGQ = 2) has generally been found sufficient. Gaussian quadrature integration, however, represents a fixed order approximation and it is not entirely satisfactory for performing integrations in the ϕ-direction since the integrands of the ψ and U functions may vary rapidly as a function of ϕ. We have therefore chosen to perform integrations in the ϕ-direction using adaptive trapezoidal rule integration (subroutine TRPADP). The adaptive numerical integration is terminated when the relative error between the two most recent

results is less than 1%. The adaptive integration procedure assures that the integrations in the ϕ-direction will be performed accurately regardless of the body size or Fourier component considered. For self-terms, in which the integrands of the ψ and U functions may be singular, the procedures indicated in Appendix A are followed. Any analytically evaluated portion of an integral function U is included by the function subroutine CANALY. Analytically evaluated portions of the ψ functions are included automatically in the subroutines ELLPTC and ELLPTR. For non-self terms the ψ functions are evaluated as indicated by Eq. (A.13) in order to increase the convergence rate of the adaptive integration in the ϕ-direction by smoothing the integrand.

B.2 Sample Case

A sample case is provided in this section to provide a convenient check for user implementation only. The sample run is not intended to adequately model any structure, but simply to exercise most sections of the code. The input data for the sample case follows:

```
1          0          1    2     1
1.0        4.0             1.0
90.        0.             -1.0   0.0     0.0
0.0        0.0
.25        0.0
.5         .25
.5         .5
.3         .65
.15        .65
0.0        .6
9999.      9999.
```

The resulting output of the program is presented on the following pages.

```
****************************
*                          *
*  CASE NUMBER:  1         *
*                          *
*  EXCITATION NUMBER:  1   *
*                          *
****************************
```

POINT	RHO	Z	DELTA-T	GAMMA
1	0.000000E+00	0.000000E+00		
	1.250000E-01	0.000000E+00	2.500000E-01	1.570796E+00
2	2.500000E-01	0.000000E+00		
	3.750000E-01	1.250000E-01	3.535534E-01	7.853982E-01
3	5.000000E-01	2.500000E-01		
	5.000000E-01	3.750000E-01	2.500000E-01	0.000000E+00
4	5.000000E-01	5.000000E-01		
	4.000000E-01	5.750000E-01	2.500000E-01	-9.272952E-01
5	3.000000E-01	6.500000E-01		
	2.250000E-01	6.500000E-01	1.500000E-01	-1.570796E+00
6	1.500000E-01	6.500000E-01		
	7.500000E-02	6.250000E-01	1.581139E-01	-1.892547E+00
7	0.000000E+00	6.000000E-01		

```
**************
*            *
* INPUT DATA *
*            *
**************

MAXIMUM NUMBER OF MODES TO BE USED =   2

ORDER OF GAUSSIAN QUADRATURE =   2

FREE SPACE WAVELENGTH =   1.0000000E+00

A DIELECTRIC BODY HAS BEEN ASSUMED:

     RELATIVE DIELECTRIC CONSTANT =   4.0000000E+00

     RELATIVE PERMEABILITY =   1.0000000E+00

INCIDENT FIELD DATA:

     THETA =   9.0000000E+01

     PHI =   0.0000000E+00

     E-THETA =   -1.0000000E+00

     E-PHI =   0.0000000E+00

*****************
*               *
* COMPUTED DATA *
*               *
*****************

PARAMETERS OF FREE SPACE:

    DIELECTRIC CONSTANT =   8.8541850E-12

    PERMEABILITY =   1.2566370E-06

    WAVENUMBER =   6.2831850E+00

    SPEED OF LIGHT =   2.9979250E+08

PARAMETERS OF THE DIELECTRIC BODY:

    DIELECTRIC CONSTANT =   3.5416740E-11

    PERMEABILITY =   1.2566370E-06

    WAVENUMBER =   1.2566370E+01

    SPEED OF LIGHT =   1.4989630E+08

    WAVELENGTH =   5.0000000E-01

OMEGA =   1.8836520E+09

FREQUENCY =   2.9979250E+08
```

"T" COMPONENT OF ELECTRIC CURRENT
FOR THETA DIRECTED PORTION OF INCIDENT FIELD

MODE NUMBER 0

RHO ***	Z ***	REAL TOTAL ****	IMAGINARY TOTAL *********	MAGNITUDE TOTAL *********	PHASE *****	REAL DENSITY ****	IMAGINARY DENSITY *********	MAGNITUDE DENSITY *********
0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	???	0.00000E+00	0.00000E+00	0.00000E+00
2.50000E-01	0.00000E+00	1.51233E-03	1.39029E-03	2.05428E-03	4.25926E+01	9.62777E-04	8.85089E-04	1.30779E-03
5.00000E-01	2.50000E-01	-4.55598E-03	4.88770E-03	6.68181E-03	1.32988E+02	-1.45021E-03	1.55580E-03	2.12689E-03
5.00000E-01	5.00000E-01	-3.69480E-03	3.48712E-03	5.08120E-03	1.36648E+02	-1.17609E-03	1.11030E-03	1.61740E-03
3.00000E-01	6.50000E-01	1.68047E-03	3.71937E-03	4.08138E-03	6.56858E+01	8.91517E-04	1.97319E-03	2.16524E-03
1.50000E-01	6.50000E-01	5.57119E-04	5.32093E-04	7.70393E-04	4.36838E+01	5.91122E-04	5.64569E-04	8.17412E-04
0.00000E+00	6.00000E-01	0.00000E+00	0.00000E+00	0.00000E+00	???	0.00000E+00	0.00000E+00	0.00000E+00

INV. COND. NO.: 3.3222760E-07 DETERMINANT: (3.1273050E-05 -6.4957890E-05) TIMES (10.)**(-200.0)

"PHI" COMPONENT OF ELECTRIC CURRENT
FOR THETA DIRECTED PORTION OF INCIDENT FIELD

MODE NUMBER 0

RHO ***	Z ***	REAL TOTAL ****	IMAGINARY TOTAL *********	MAGNITUDE TOTAL *********	PHASE *****	REAL DENSITY ****	IMAGINARY DENSITY *********	MAGNITUDE DENSITY *********
1.25000E-01	0.00000E+00	-7.38063E-13	3.55961E-13	8.19418E-13	1.54252E+02	-9.39731E-13	4.53223E-13	1.04332E-12
3.75000E-01	1.25000E-01	9.07931E-13	2.06482E-12	2.25552E-12	6.62642E+01	3.85338E-13	8.76336E-13	9.57314E-13
5.00000E-01	3.75000E-01	6.34647E-12	4.97980E-13	6.36597E-12	4.48656E+00	2.02014E-12	1.58512E-13	2.02635E-12
4.00000E-01	5.75000E-01	-9.43097E-12	3.42090E-12	1.00322E-11	1.60063E+02	-3.75246E-12	1.36113E-12	3.99170E-12
2.25000E-01	6.50000E-01	-5.21204E-12	-5.92272E-12	7.88948E-12	-1.31348E+02	-3.68676E-12	-4.18946E-12	5.58066E-12
7.50000E-02	6.25000E-01	-8.71204E-13	-2.67353E-12	2.81190E-12	-1.08049E+02	-1.84875E-12	-5.67342E-12	5.96704E-12

INV. COND. NO.: 3.3222760E-07 DETERMINANT: (3.1273050E-05 -6.4957890E-05) TIMES (10.)**(-200.0)

"T" COMPONENT OF MAGNETIC CURRENT
FOR THETA DIRECTED PORTION OF INCIDENT FIELD

MODE NUMBER 0

RHO ***	Z ***	REAL TOTAL ****	IMAGINARY TOTAL *********	MAGNITUDE TOTAL *********	PHASE *****	REAL DENSITY ****	IMAGINARY DENSITY *********	MAGNITUDE DENSITY *********
0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	???	0.00000E+00	0.00000E+00	0.00000E+00
2.50000E-01	0.00000E+00	-2.37470E-09	-3.39079E-09	4.13964E-09	-1.25005E+02	-1.51178E-09	-2.15864E-09	2.63538E-09
5.00000E-01	2.50000E-01	8.85322E-10	-2.63334E-09	2.77818E-09	-7.14175E+01	2.81807E-10	-8.38220E-10	8.84323E-10
5.00000E-01	5.00000E-01	-4.24101E-09	2.56733E-09	4.95755E-09	1.48811E+02	-1.34995E-09	8.17205E-10	1.57804E-09
3.00000E-01	6.50000E-01	-6.20603E-12	-8.37083E-10	8.37106E-10	-9.04248E+01	-3.29240E-12	-4.44086E-10	4.44099E-10
1.50000E-01	6.50000E-01	7.50407E-10	-1.08616E-09	1.32017E-09	-5.53601E+01	7.96206E-10	-1.15245E-09	1.40074E-09
0.00000E+00	6.00000E-01	0.00000E+00	0.00000E+00	0.00000E+00	???	0.00000E+00	0.00000E+00	0.00000E+00

INV. COND. NO.: 3.3222760E-07 DETERMINANT: (3.1273050E-05 -6.4957890E-05) TIMES (10.)**(-200.0)

"PHI" COMPONENT OF MAGNETIC CURRENT
FOR THETA DIRECTED PORTION OF INCIDENT FIELD

MODE NUMBER 0

RHO ***	Z ***	REAL TOTAL ****	IMAGINARY TOTAL *********	MAGNITUDE TOTAL *********	PHASE *****	REAL DENSITY ****	IMAGINARY DENSITY *********	MAGNITUDE DENSITY *********
1.25000E-01	0.00000E+00	1.27316E-01	1.80166E-01	2.20610E-01	5.47527E+01	1.62103E-01	2.29394E-01	2.80890E-01
3.75000E-01	1.25000E-01	3.45628E-01	6.69916E-02	3.52060E-01	1.09694E+01	1.46689E-01	2.84321E-02	1.49419E-01
5.00000E-01	3.75000E-01	-1.40593E-01	-6.24160E-01	6.39798E-01	-1.02694E+02	-4.47521E-02	-1.98676E-01	2.03654E-01
4.00000E-01	5.75090E-01	6.48447E-02	-1.02929E-01	1.21652E-01	-5.77893E+01	2.58009E-02	-4.09542E-02	4.84039E-02
2.25000E-01	6.50000E-01	7.42488E-02	1.83489E-01	1.97942E-01	6.79693E+01	5.25203E-02	1.29792E-01	1.40015E-01
7.50000E-02	6.25000E-01	3.42808E-02	1.05092E-01	1.10542E-01	7.19339E+01	7.27461E-02	2.23013E-01	2.34578E-01

INV. COND. NO.: 3.3222760E-07 DETERMINANT: (3.1273050E-05 -6.4957890E-05) TIMES (10.)**(-200.0)

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"T" COMPONENT OF ELECTRIC CURRENT
FOR THETA DIRECTED PORTION OF INCIDENT FIELD

MODE NUMBER 1

COS(PHI) COEFFICIENTS

RHO ***	Z ***	REAL TOTAL ****	IMAGINARY TOTAL *********	MAGNITUDE TOTAL *********	PHASE *****	REAL DENSITY ****	IMAGINARY DENSITY *********	MAGNITUDE DENSITY *********
0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	???	???	???	???
2.50000E-01	0.00000E+00	1.52079E-03	1.01818E-03	1.83017E-03	3.38026E+01	9.68168E-04	6.48195E-04	1.16512E-03
5.00000E-01	2.50000E-01	2.82837E-04	-1.28673E-03	1.31745E-03	-7.76030E+01	9.00297E-05	-4.09579E-04	4.19357E-04
5.00000E-01	5.00000E-01	-1.26725E-03	-9.66056E-04	1.59348E-03	-1.42681E+02	-4.03378E-04	-3.07505E-04	5.07221E-04
3.00000E-01	6.50000E-01	1.65503E-03	1.51015E-03	2.24046E-03	4.23793E+01	8.78019E-04	8.01160E-04	1.18860E-03
1.50000E-01	6.50000E-01	7.63971E-04	-6.54471E-07	7.63972E-04	-4.90836E-02	8.10599E-04	-6.94416E-07	8.10599E-04
0.00000E+00	6.00000E-01	0.00000E+00	0.00000E+00	0.00000E+00	???	???	???	???

INV. COND. NO.: 2.6376860E-07 DETERMINANT: (4.0919200E-06 -5.6265210E-06) TIMES (10.)**(-200.0)

ROOT-MEAN-SQUARE ERROR AT THIS STAGE: 3.9475950E-01

"PHI" COMPONENT OF ELECTRIC CURRENT
FOR THETA DIRECTED PORTION OF INCIDENT FIELD

MODE NUMBER 1

SIN(PHI) COEFFICIENTS

RHO ***	Z ***	REAL TOTAL ****	IMAGINARY TOTAL *********	MAGNITUDE TOTAL *********	PHASE *****	REAL DENSITY ****	IMAGINARY DENSITY *********	MAGNITUDE DENSITY *********
1.25000E-01	0.00000E+00	-1.49471E-03	-5.80045E-04	1.60331E-03	-1.58790E+02	-1.90313E-03	-7.38537E-04	2.04140E-03
3.75000E-01	1.25000E-01	-6.28147E-04	1.80451E-04	6.53553E-04	1.63972E+02	-2.66594E-04	7.65858E-05	2.77376E-04
5.00000E-01	3.75000E-01	-4.06397E-04	-8.65970E-04	9.56589E-04	-1.15141E+02	-1.29360E-04	-2.75647E-04	3.04492E-04
4.00000E-01	5.75000E-01	9.08867E-04	-6.23586E-04	1.10222E-03	-3.44546E+01	3.61627E-04	-2.48117E-04	4.38561E-04
2.25000E-01	6.50000E-01	1.50175E-03	-8.15038E-06	1.50177E-03	-3.10956E-01	1.06227E-03	-5.76521E-06	1.06228E-03
7.50000E-02	6.25000E-01	-7.09715E-05	-4.15860E-04	4.21085E-04	-9.97032E+01	-1.50606E-04	-8.80786E-04	8.93569E-04

INV. COND. NO.: 2.6376860E-07 DETERMINANT: (4.0919200E-06 -5.6265210E-06) TIMES (10.)**(-200.0)

ROOT-MEAN-SQUARE ERROR AT THIS STAGE: 1.0000000E+00

"T" COMPONENT OF MAGNETIC CURRENT
FOR THETA DIRECTED PORTION OF INCIDENT FIELD

MODE NUMBER 1

SIN(PHI) COEFFICIENTS

RHO ***	Z ***	REAL TOTAL ****	IMAGINARY TOTAL *********	MAGNITUDE TOTAL *********	PHASE *****	REAL DENSITY ****	IMAGINARY DENSITY *********	MAGNITUDE DENSITY *********
0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	???	???	???	???
2.50000E-01	0.00000E+00	-2.98247E-01	-1.74145E-01	3.45366E-01	-1.49720E+02	-1.89870E-01	-1.10864E-01	2.19867E-01
5.00000E-01	2.50000E-01	-4.01491E-01	4.57507E-01	6.08693E-01	1.31269E+02	-1.27799E-01	1.45629E-01	1.93753E-01
5.00000E-01	5.00000E-01	4.94885E-01	-7.92825E-01	9.34603E-01	-5.80274E+01	1.57527E-01	-2.52364E-01	2.97493E-01
3.00000E-01	6.50000E-01	2.46446E-01	4.18585E-01	4.85746E-01	5.95122E+01	1.30744E-01	2.22066E-01	2.57696E-01
1.50000E-01	6.50000E-01	-6.07100E-01	5.20082E-01	7.99410E-01	1.39414E+02	-6.44154E-01	5.51824E-01	8.48200E-01
0.00000E+00	6.00000E-01	0.00000E+00	0.00000E+00	0.00000E+00	???	???	???	???

INV. COND. NO.: 2.6376860E-07 DETERMINANT: (4.0919200E-06 -5.6265210E-06) TIMES (10.)**(-200.0)

ROOT-MEAN-SQUARE ERROR AT THIS STAGE: 1.0000000E+00

"PHI" COMPONENT OF MAGNETIC CURRENT
FOR THETA DIRECTED PORTION OF INCIDENT FIELD

MODE NUMBER 1

COS(PHI) COEFFICIENTS

RHO ***	Z ***	REAL TOTAL ****	IMAGINARY TOTAL *********	MAGNITUDE TOTAL *********	PHASE *****	REAL DENSITY ****	IMAGINARY DENSITY *********	MAGNITUDE DENSITY *********
1.25000E-01	0.00000E+00	4.54932E-01	-2.53718E-01	5.20899E-01	-2.91486E+01	5.79237E-01	-3.23044E-01	6.63229E-01
3.75000E-01	1.25000E-01	-1.28526E+00	1.59347E+00	2.04721E+00	1.28889E+02	-5.45483E-01	6.76290E-01	8.68862E-01
5.00000E-01	3.75000E-01	-1.90686E+00	2.34260E+00	3.02058E+00	1.29145E+02	-6.06974E-01	7.45673E-01	9.61481E-01
4.00000E-01	5.75000E-01	-1.20882E+00	1.42639E+00	1.86972E+00	1.30280E+02	-4.80974E-01	5.67543E-01	7.43936E-01
2.25000E-01	6.50000E-01	3.13761E-02	3.14860E-01	3.16420E-01	8.43092E+01	2.21941E-02	2.22718E-01	2.23821E-01
7.50000E-02	6.25000E-01	2.69081E-01	-1.95207E-01	3.32431E-01	-3.59593E+01	5.71008E-01	-4.14242E-01	7.05441E-01

INV. COND. NO.: 2.6376860E-07 DETERMINANT: (4.0919200E-06 -5.6265210E-06) TIMES (10.)**(-200.0)

ROOT-MEAN-SQUARE ERROR AT THIS STAGE: 9.8722930E-01

"T" COMPONENT OF ELECTRIC CURRENT
FOR THETA DIRECTED PORTION OF INCIDENT FIELD

SUM OF MODES 0 THROUGH 1

OBSERVED AT PHI = 0.0 DEGREES

RHO ***	Z ***	REAL TOTAL ****	IMAGINARY TOTAL *********	MAGNITUDE TOTAL *********	PHASE *****	REAL DENSITY ****	IMAGINARY DENSITY *********	MAGNITUDE DENSITY *********
0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	???	???	???	???
2.50000E-01	0.00000E+00	3.03312E-03	2.40848E-03	3.87306E-03	3.84516E+01	1.93094E-03	1.53328E-03	2.46567E-03
5.00000E-01	2.50000E-01	-4.27315E-03	3.62097E-03	5.58809E-03	1.39879E+02	-1.36018E-03	1.14622E-03	1.77874E-03
5.00000E-01	5.00000E-01	-4.96205E-03	2.52207E-03	5.56622E-03	1.53057E+02	-1.57947E-03	8.02799E-04	1.77178E-03
3.00000E-01	6.50000E-01	3.33550E-03	5.22952E-03	6.20269E-03	5.74694E+01	1.76954E-03	2.77435E-03	3.29063E-03
1.50000E-01	6.50000E-01	1.32109E-03	5.31439E-04	1.42398E-03	2.19136E+01	1.40172E-03	5.63874E-04	1.51089E-03
0.00000E+00	6.00000E-01	0.00000E+00	0.00000E+00	0.00000E+00	???	???	???	???

ROOT-MEAN-SQUARE ERROR AT THIS STAGE: 3.9475050E-01 .

"PHI" COMPONENT OF ELECTRIC CURRENT
FOR THETA DIRECTED PORTION OF INCIDENT FIELD

SUM OF MODES 0 THROUGH 1

OBSERVED AT PHI = 90.0 DEGREES

RHO ***	Z ***	REAL TOTAL ****	IMAGINARY TOTAL *********	MAGNITUDE TOTAL *********	PHASE *****	REAL DENSITY ****	IMAGINARY DENSITY *********	MAGNITUDE DENSITY *********
1.25000E-01	0.00000E+00	-1.49471E-03	-5.80045E-04	1.60331E-03	-1.58790E+02	-1.90312E-03	-7.38537E-04	2.04140E-03
3.75000E-01	1.25000E-01	-6.28147E-04	1.80451E-04	6.53553E-04	1.63972E+02	-2.66594E-04	7.65858E-05	2.77376E-04
5.00000E-01	3.75000E-01	-4.06397E-04	-8.65970E-04	9.56589E-04	-1.15141E+02	-1.29360E-04	-2.75647E-04	3.04492E-04
4.00000E-01	5.75000E-01	9.08867E-04	-6.23586E-04	1.10222E-03	-3.44546E+01	3.61627E-04	-2.48117E-04	4.38561E-04
2.25000E-01	6.50000E-01	1.50175E-03	-8.15038E-06	1.50177E-03	-3.10957E-01	1.06227E-03	-5.76522E-06	1.06228E-03
7.50000E-02	6.25000E-01	-7.09715E-05	-4.15060E-04	4.21085E-04	-9.97032E+01	-1.50606E-04	-8.80786E-04	8.93569E-04

ROOT-MEAN-SQUARE ERROR AT THIS STAGE: 1.0000000E+00

"T" COMPONENT OF MAGNETIC CURRENT
FOR THETA DIRECTED PORTION OF INCIDENT FIELD

SUM OF MODES 0 THROUGH 1

OBSERVED AT PHI = 90.0 DEGREES

RHO ***	Z ***	REAL TOTAL ****	IMAGINARY TOTAL *********	MAGNITUDE TOTAL *********	PHASE *****	REAL DENSITY ****	IMAGINARY DENSITY *********	MAGNITUDE DENSITY *********
0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	???	???	???	???
2.50000E-01	0.00000E+00	-2.98247E-01	-1.74145E-01	3.45366E-01	-1.49720E+02	-1.89870E-01	-1.10864E-01	2.19867E-01
5.00000E-01	2.50000E-01	-4.01491E-01	4.57507E-01	6.08693E-01	1.31269E+02	-1.27799E-01	1.45629E-01	1.93753E-01
5.00000E-01	5.00000E-01	4.94885E-01	-7.92825E-01	9.34603E-01	-5.80274E+01	1.57527E-01	-2.52364E-01	2.97493E-01
3.00000E-01	6.50000E-01	2.46446E-01	4.18585E-01	4.85746E-01	5.95122E+01	1.30744E-01	2.22066E-01	2.57696E-01
1.50000E-01	6.50000E-01	-6.07100E-01	5.20082E-01	7.99410E-01	1.39414E+02	-6.44154E-01	5.51824E-01	8.48200E-01
0.00000E+00	6.00000E-01	0.00000E+00	0.00000E+00	0.00000E+00	???	???	???	???

ROOT-MEAN-SQUARE ERROR AT THIS STAGE: 1.0000000E+00

"PHI" COMPONENT OF MAGNETIC CURRENT
FOR THETA DIRECTED PORTION OF INCIDENT FIELD

SUM OF MODES 0 THROUGH 1

OBSERVED AT PHI = 0.0 DEGREES

RHO ***	Z ***	REAL TOTAL ****	IMAGINARY TOTAL *********	MAGNITUDE TOTAL *********	PHASE *****	REAL DENSITY ****	IMAGINARY DENSITY *********	MAGNITUDE DENSITY *********
1.25000E-01	0.00000E+00	5.82248E-01	-7.35522E-02	5.86875E-01	-7.19973E+00	7.41341E-01	-9.36496E-02	7.47232E-01
3.75000E-01	1.25000E-01	-9.39637E-01	1.66046E+00	1.90789E+00	1.19505E+02	-3.98794E-01	7.04722E-01	8.09735E-01
5.00000E-01	3.75000E-01	-2.04746E+00	1.71844E+00	2.67304E+00	1.39993E+02	-6.51726E-01	5.46996E-01	8.50854E-01
4.00000E-01	5.75000E-01	-1.14397E+00	1.32346E+00	1.74935E+00	1.30839E+02	-4.55173E-01	5.26589E-01	6.96045E-01
2.25000E-01	6.50000E-01	1.05625E-01	4.98349E-01	5.09420E-01	7.80333E+01	7.47143E-02	3.52510E-01	3.60341E-01
7.50000E-02	3.25000E-01	3.03362E-01	-9.01147E-02	3.16464E-01	-1.65442E+01	6.43755E-01	-1.91229E-01	6.71557E-01

ROOT-MEAN-SQUARE ERROR AT THIS STAGE: 9.8722930E-01

B.3 Program Listing

```
C                        PROGRAM DBR
C
C                        VERSION 4.01
C
C                        JULY 20, 1978
C***********************************************************************
C
C      PROGRAM DBR CALCULATES THE CURRENTS (TOTAL AND DENSITY) FOR A
C      BODY OF REVOLUTION, EITHER DIELECTRIC OR PERFECTLY CONDUCTING.
C
C
C      PROGRAM "MAIN." READS ALL THE INPUT DATA EXCEPT FOR DATA
C      CONCERNING THE GENERATING CURVE OF THE BODY OF REVOLUTION.
C      SUBROUTINE "GENCUR" IS CALLED TO READ THIS DATA.  "MAIN." THEN
C      COMPUTES THE DIMENSIONS NECESSARY TO RUN THE PROGRAM FOR THE CASE
C      IN QUESTION AND ABORTS EXECUTION OF THE CASE IF THE ABSOLUTE
C      DIMENSIONS PROVIDED IN "MAIN." ARE INSUFFICIENT.  IF THE
C      DIMENSIONS ARE ADEQUATE "MAIN." CALLS SUBROUTINE "SLTN" TO
C      SOLVE THE PROBLEM.
C
C       AUXILIARY ROUTINES NEEDED:
C
C          CGQ1   - INTEGRATION OF A COMPLEX FUNCTION BY GAUSSIAN QUAD.
C          TRPADP - INTEGRATION OF A COMPLEX FUNCTION BY ADAPTIVE
C                   TRAPEZOIDAL RULE
C          CSMINV - INVERSION OF A COMPLEX MATRIX
C          ICRMUL - COMPLEX MATRIX MULTIPLICATION
C          BESSEL - COMPUTATION OF BESSEL FUNCTION OF ARBITRARY ORDER
C          ELIC1K - COMPUTES COMPLETE ELLIPTIC INTEGRAL OF FIRST KIND
C
C       ASSIGNMENTS:
C
C          READ FROM DSK: #2
C          WRITE PRINTED OUTPUT ON LPT: #5 THROUGH #15
C           (AS NECESSARY:  FOR MULTIPLE EXCITATIONS)
C
C
C***********************************************************************
      IMPLICIT COMPLEX (C)
      REAL LAMBDA,MU,MUR
C "LOCATE 1"
      DIMENSION C(10296),R(306)
      COMMON/MBE/MODEB,MODEE
      COMMON/PNT/NPTS,NUNKT,NUNKPH,NUNK2
      COMMON/INT/NMODEM,IDB,NGQ
      COMMON/WVL/LAMBDA,EPSR,MUR
      COMMON/CAS/ICASE
C "LOCATE 2"
      NDMSNC=10296
      NDMSNR=306
C
```

```
C-----NOTE:  WHEN THE DIMENSION OF "C" OR "R" IS CHANGED, THE VALUE OF
C            "NDMSNC" OR "NDMSNR", RESPECTIVELY, MUST BE CHANGED TO THE
C            APPROPRIATE VALUE FOR PROGRAM OPERATION.
C
C     INPUT DATA DESCRIPTION:
C       (ALL DATA IS READ IN "FREE" FORMAT)
C
C       NFLDS  = NUMBER OF DIFFERENT EXCITATIONS TO BE SOLVED FOR
C                THIS STRUCTURE.
C
C       MODEB  = STARTING FOURIER COMPONENT NUMBER
C
C       MODEE  = ENDING FOURIER COMPONENT NUMBER
C
C       IDB    = A CONTROL VARIABLE:
C              = 1,  IF THE STRUCTURE IS A DIELECTRIC BODY
C              = 0,  IF THS STRUCTURE IS A PERFECT CONDUCTOR
C
C       ANTFD  = NUMBER OF SUBDOMAIN WHICH IS FED BY A DELTA GAP SOURCE.
C                IF  =0,  THERE IS NO SOURCE.   THE SECOND COORDINATE
C                POINT READ IN IS CONSIDERED TO BE THE FIRST SUBDOMAIN,
C                AND SO ON.
C
C       EPSR   = RELATIVE DIELECTRIC CONSTANT OF THE BODY
C                (IF DIELECTRIC)
C
C       MUR    = RELATIVE PERMEABILITY OF THE BODY (IF DIELECTRIC)
C
C       LAMBDA = WAVELENGTH (IN METERS) IN FREE SPACE
C
C       NGQ    = ORDER OF GAUSSIAN QUADRATURE INTEGRATION TO BE USED
C
C       THETA  = SPHERICAL COORDINATE ANGLE OF INCIDENCE (IN DEGREES)
C
C       PHI    = SPHERICAL COORDINATE ANGLE OF INCIDENCE (IN DEGREES)
C
C       ETHETA = SIGNED MAGNITUDE OF THE E-THETA INCIDENT FIELD
C
C       EPHI   = SIGNED MAGNITUDE OF THE E-PHI INCIDENT FIELD
C
C
      DO 40 ICASE = 1, 50
      READ(2,10000,END=50)NFLDS,MODEB,MODEE,NGQ,IDB
      READ(2,10001) LAMBDA, EPSR, MUR
      NMODEM=MODEE-MODEB+1
      I2=NFLDS
      I3=NFLDS+I2
      I4=NFLDS+I3
      I5=NFLDS+I4
      I6=NFLDS+I5
      DO 10 I = 1, NFLDS
      READ(2,10001) THETA, PHI, ETHETA, EPHI, ANTFD
      R(I)=THETA
      R(I2+I)=PHI
      R(I3+I)=ETHETA
      R(I4+I)=EPHI
      R(I5+I)=ANTFD
   10 CONTINUE
      CALL GENCUR(R(I6+1),I6,NDMSNR,NPTS,NREQR)
      NREQR=NREQR+I6
      NUNKT=NPTS-2
```

```
      NUNKPH=NPTS-1
      IF(IDB .EQ. 0) GO TO 20
      NUNKT=NUNKT*2
      NUNKPH=NUNKPH*2
   20 NUNK=NUNKT+NUNKPH
      NUNK2=NUNK*NUNK
      NREQC=NUNK*NFLDS*6+NUNK2
      IF( NREQC .GT. NDMSNC .OR. NREQR .GT. NDMSNR) GO TO 30
      NPTSM1=NPTS-1
      MR2=NFLDS+1
      MR3=NFLDS+MR2
      MR4=NFLDS+MR3
      MR5=NFLDS+MR4
      MR6=NFLDS+MR5
      MR7=NPTSM1+MR6
      MR8=NPTSM1+MR7
      MR9=NPTSM1+MR8
      MR10=NPTSM1+MR9
      MR11=NPTS+MR10
C-----MR12=NPTS+MR11
      MC2=NUNK2+1
      MC3=NUNK*NFLDS+MC2
      MC4=NUNK*NFLDS+MC3
      MC5=NUNK*NFLDS+MC4
      MC6=NUNK*NFLDS+MC5
C-----MC7=NUNK*NFLDS+MC6
      CALL ZERO(C,NREQC)
      CALL SLTN(
     $           C(1),C(MC2),C(MC3),C(MC4),C(MC5),C(MC6),
     $           R(1),R(MR2),R(MR3),R(MR4),R(MR5),R(MR6),R(MR7),R(MR8),
     $           R(MR9),R(MR10),R(MR11),NUNK,NFLDS                          )
      GO TO 40
   30 CONTINUE
      WRITE(5,10002) ICASE, NDMSNC, NDMSNR, NREQC, NREQR
   40 CONTINUE
   50 CONTINUE
10000 FORMAT(9I)
10001 FORMAT(9E)
10002 FORMAT(' CASE NUMBER'I3,' EXECUTION ABORTED:  DIMENSIONS INSUFFIC
     #ENT'//' DIMENSION GIVEN IN PROGRAM:'//5X,'C('I6,')'5X,'R('I6,')'//
     #' DIMENSIONS REQUIRED FOR THIS CASE:'//5X,'C('I6,')'5X,'R('I6,')')
      STOP
      END
      SUBROUTINE GENCUR(S,NFLDS4,NDMSNS,NPTS,NREQR)
C***********************************************************************
C
C     SUBROUTINE "GENCUR" READS THE GENERATING CURVE DATA FOR THE BODY
C     OF REVOLUTION AND USES THIS INFORMATION TO COMPUTE THE DIMENSIONS
C     NECESSARY OF THE REAL VECTOR "S".  IT ALSO USES THE GENERATING
C     CURVE DATA TO COMPUTE INTERMEDIATE COORDINATE LOCATIONS, VALUES
C     OF STEP SIZE, AND ANGLES CORRESPONDING TO THE DIRECTION OF EACH
C     ELEMENTAL SURFACE.
C
C***********************************************************************
      DIMENSION S(1)
      NDMSN=NDMSNS-NFLDS4
C
C     INPUT DATA DESCRIPTION:
C
C       RHO  = CYLINDRICAL COORDINATE RHO (IN METERS)
C
```

```
C       Z     = CYLINDRICAL COORDINATE Z (IN METERS)
C
C     NOTE:  THE LAST RECORD OF THE GENERATING CURVE DATA MUST BE
C              9999.   9999.
C     (THE SPACING IS ARBITRARY).
C
C
      I=1
      DO 10 K = 1, NDMSN
   10 S(K)=0.0
   20 READ(2,10000) RHO, Z
      NPTS=I-1
      IF(RHO .EQ. 9999. .AND. Z .EQ. 9999.) GO TO 30
      K=2*I
      IF(K .GT. NDMSN) GO TO 20
      S(K-1)=RHO
      S(K)=Z
      I=I+1
      GO TO 20
C
C-----COMPUTE THE DIMENSION OF "S" REQUIRED FOR THIS CASE
C
   30 NREQR=6*NPTS-4
      IF(NREQR .GT. NDMSN) RETURN
      NSTRT=4*NPTS-4
C
C-----REORDER THE INPUT DATA
C
      DO 40 I = 1, NPTS
      K=2*I
      NI=NSTRT+I
      S(NI)=S(K-1)
      S(NPTS+NI)=S(K)
   40 CONTINUE
      NPTSM1=NPTS-1
      DO 50 I = 1, NPTSM1
      NI=NSTRT+I
      RI=S(NI)
      RIP1=S(NI+1)
      NI=NPTS+NI
      ZI=S(NI)
      ZIP1=S(NI+1)
C
C-----STORE RHO AND Z HALFWAY POINTS
C
      S(I)=(RI+RIP1)/2.0
      S(I+NPTSM1)=(ZI+ZIP1)/2.0
      DIIP1=SQRT((RIP1-RI)**2+(ZIP1-ZI)**2)
      ARGN=(RIP1-RI)/DIIP1
      ARGD=(ZIP1-ZI)/DIIP1
C
C-----STORE DELTA-T VECTOR
C
      S(I+2*NPTSM1)=DIIP1
C
C-----STORE GAMMA VECTOR
C
      S(I+3*NPTSM1)=ATAN2(ARGN,ARGD)
   50 CONTINUE
10000 FORMAT(9E)
      RETURN
```

```
      END
      SUBROUTINE SLTN(
     $                 CZ,CIT,CIP,CVT,CVP,CITOLD,
     $                 THETA,PHI,ETHETA,EPHI,ANTFD,RHOPH,ZPH,DELTAT,
     $                 GAMMA,RHO,Z,NUNK,NFLDS                     )
C**********************************************************************
C
C     SUBROUTINE "SLTN" COMPUTES AND OUTPUTS THE RESULTS.
C       PRINTED RESULTS ARE OUTPUT TO DEVICES NUMBER 5 THROUGH 15.
C        (FOR MULTIPLE EXCITATIONS, EACH CASE IS OUTPUT TO A NEW
C        DEVICE #, BEGINNING WITH DEVICE #5)
C
C**********************************************************************
      IMPLICIT COMPLEX (C)
      REAL LAMBDA,MUR,MU1,MU2
      DIMENSION POL(2),FTC(2)
      DIMENSION CZ(NUNK,NUNK),CIT(NUNK,NFLDS),CIP(NUNK,NFLDS)
      DIMENSION CVT(NUNK,NFLDS),CVP(NUNK,NFLDS)
      DIMENSION CITOLD(NUNK,NFLDS,2)
      DIMENSION THETA(1),PHI(1),ETHETA(1),EPHI(1),ANTFD(1)
      DIMENSION RHO(1),Z(1),RHOPH(1),ZPH(1),DELTAT(1),GAMMA(1)
      COMMON/MBE/MODEB,MODEE
      COMMON/SCAFAC/ISCALE
      COMMON/PNT/NPTS,NUNKT,NUNKPH,NUNK2
      COMMON/INT/NMODEM,IDB,NGQ
      COMMON/WVL/LAMBDA,EPSR,MUR
      COMMON/CAS/ICASE
      COMMON/FRQ/PI,AK1,AK2,SL1,SL2,OMEGA
      COMMON/PRM/EPS1,EPS2,MU1,MU2
      DATA POL/'THETA',' PHI '/
      DATA FTC/'SIN','COS',
      NPTSM1=NPTS-1
      ZER=0.0
      PI=3.1415926536
      AK1=2.0*PI/LAMBDA
      SL1=2.997925E8
      OMEGA=AK1*SL1
      FREQ=OMEGA/2.0/PI
      MU1=PI*4.0E-7
      EPS1=1.0/(MU1*SL1*SL1)
      IF(IDB .EQ. 0) GO TO 10
      EPS2=EPS1*EPSR
      MU2=MU1*MUR
      SL2=1.0/SQRT(MU2*EPS2)
      AK2=OMEGA/SL2
      WVL2=2.0*PI/AK2
   10 CONTINUE
C
C-----PARAMETER PRINT-OUT OPTION
C
C       GO TO 50
C
      DO 40 JJ=1,NFLDS
      JW=JJ+4
      WRITE(JW,10036) ICASE, JJ
      WRITE(JW,10000)
      DO 30 J=1,NPTS
      WRITE(JW,10001) J, RHO(J), Z(J)
      IF(J .EQ. NPTS) GO TO 20
      WRITE(JW,10002) RHOPH(J), ZPH(J), DELTAT(J), GAMMA(J)
   20 CONTINUE
```

```
      IF(NFLDS .GT. 11) WRITE(JW,10033)
   30 CONTINUE
   40 CONTINUE
   50 CONTINUE
      DO 60 J=1,NFLDS
      JW=J+4
      WRITE(JW,10003)
      WRITE(JW,10004) NMODEM, NGQ
      WRITE(JW,10005) LAMBDA
      IF(IDB .EQ. 0) WRITE(JW,10006)
      IF(IDB .EQ. 1) WRITE(JW,10007) EPSR, MUR
      WRITE(JW,10008) THETA(J), PHI(J), ETHETA(J), EPHI(J)
      IANTFD=IFIX(ANTFD(J))
      IF(IANTFD .NE. 0) WRITE(JW,10025) RHO(IANTFD+1),Z(IANTFD+1)
      WRITE(JW,10017)
      WRITE(JW,10022)
      WRITE(JW,10018) EPS1, MU1, AK1, SL1
      IF(IDB .EQ. 1) WRITE(JW,10019)
      IF(IDB .EQ. 1) WRITE(JW,10018) EPS2, MU2, AK2, SL2
      IF(IDB .EQ. 1) WRITE(JW,10023) WVL2
      WRITE(JW,10020) OMEGA, FREQ
   60 CONTINUE
      M0ONLY=1
      M1ONLY=1
      DO 70 JJ=1,NFLDS
      IF(EPHI(JJ) .NE. 0.0 .OR. ETHETA(JJ) .NE. 0.0) M0ONLY=0
   70 IF(THETA(JJ) .NE. 0.0 .AND. THETA(JJ) .NE. 180.0) M1ONLY=0
      IF(M1ONLY .EQ. 1) NMODEM=1
      IF(M0ONLY .EQ. 1) NMODEM=1
      DO 340 JM=1,NMODEM
      MODE=JM-1+MODEB
      IF(M1ONLY .EQ. 1) MODE=1
      IF(M0ONLY .EQ. 1) MODE=0
      IF(JM .NE. 1) CALL ZERO(CZ,NUNK*NUNK)
      CALL ZMATRX(CZ,RHO,Z,RHOPH,ZPH,DELTAT,GAMMA,NUNK,MODE)
C
C?????????????????????????????????????????????????????????????????????????????
C                MATRIX PRINTOUT OPTION
C?????????????????????????????????????????????????????????????????????????????
C
C       WRITE(5,11200) (((II,JJ,CZ(II,JJ)),JJ=1,NUNK),II=1,NUNK)
C
      CALL CVFILL(CVT,CVP,RHO,Z,RHOPH,ZPH,DELTAT,GAMMA,THETA,PHI,
     #            ETHETA,EPHI,ANTFD,NUNK,NFLDS,MODE)
C
C?????????????????????????????????????????????????????????????????????????????
C                DRIVE MATRIX PRINTOUT OPTION
C?????????????????????????????????????????????????????????????????????????????
C
C       WRITE(5,11100) ((CVT(I,1),CVP(I,1)),I=1,NUNK)
C
      ISCALE=0
      CALL CSMINV(CZ,NUNK,NUNK,CDETRM,ACOND,IERR)
C
C?????????????????????????????????????????????????????????????????????????????
C                INVERSE MATRIX PRINTOUT OPTION
C?????????????????????????????????????????????????????????????????????????????
C
C       WRITE(5,11200) (((II,JJ,CZ(II,JJ)),JJ=1,NUNK),II=1,NUNK)
C
      POWER=-FLOAT(ISCALE)*10.
```

```
      ITHETA=1
      CALL ICRMUL(CZ,CVT,CIT,NUNK,NUNK,NUNK,NFLDS)
      CALL SUMOUT(CIT,NUNK,NFLDS,MODE,ITHETA)
      ITHETA=0
      CALL ICRMUL(CZ,CVT,CIP,NUNK,NUNK,NUNK,NFLDS)
      CALL SUMOUT(CIP,NUNK,NFLDS,MODE,ITHETA)
   80 CONTINUE
      CZERO=CMPLX(0.0,0.0)
      NMODE1=NMODEM-1
C
C-----CURRENT NORMALIZATION OPTION
C
C       CALL NORMAL(CIT,ETHETA,EPHI,NUNK,NFLDS)
C       CALL NORMAL(CIP,ETHETA,EPHI,NUNK,NFLDS)
C
   90 CONTINUE
      NTE=NUNKT
      IF(IDB .EQ. 1) NTE=NTE/2
      NPHE=NTE+1
      DO 330 J=1,NFLDS
      JW=J+4
      M0PR=0
      M1PR=0
      IF(EPHI(J) .EQ. 0. .AND. ETHETA(J) .EQ. 0.) M0PR=1
      IF(THETA(J) .EQ. 0. .OR. THETA(J) .EQ. 180.) M1PR=1
      IF(M0PR .EQ. 1) M1PR=0
      IF(M0PR .EQ. 1) GO TO 110
      IF(M1PR .EQ. 1) GO TO 110
      DO 100 JSUM=1,NUNK
      CVT(JSUM,J)=CITOLD(JSUM,J,1)
      CVP(JSUM,J)=CITOLD(JSUM,J,2)
      OB1=1.0
      OB2=SIN(FLOAT(MODE)*3.1415926/2.0)
      IF(MODE .EQ. 0) OB2=1.0
      OBSERV=OB1
      IF(JSUM .GT. NTE) OBSERV=OB2
      IF(JSUM .GT. 2*NTE+NPHE) OBSERV=OB1
      CITOLD(JSUM,J,1)=CITOLD(JSUM,J,1)+CIT(JSUM,J)*CMPLX(OBSERV,0.)
      OBSERV=OB2
      IF(JSUM .GT. NTE) OBSERV=OB1
      IF(JSUM .GT. 2*NTE+NPHE) OBSERV=OB2
      CITOLD(JSUM,J,2)=CITOLD(JSUM,J,2)+CIP(JSUM,J)*CMPLX(OBSERV,0.)
  100 CONTINUE
      IF(MODE .EQ. 0) GO TO 110
      LOC2=NTE+1
      CALL RMSERR(CITOLD(1,J,1),CVT(1,J),NTE,ETTPC)
      CALL RMSERR(CITOLD(LOC2,J,1),CVT(LOC2,J),NPHE,EPTPC)
      CALL RMSERR(CITOLD(1,J,2),CVP(1,J),NTE,ETPPC)
      CALL RMSERR(CITOLD(LOC2,J,2),CVP(LOC2,J),NPHE,EPPPC)
      IF(IDB .EQ. 0) GO TO 110
      LOC1=LOC2+NPHE
      LOC2=LOC1+N[illegible]
      CALL RMSERR(CITOLD(LOC1,J,1),CVT(LOC1,J),NTE,ETTDI)
      CALL RMSERR(CITOLD(LOC2,J,1),CVT(LOC2,J),NPHE,EPTDI)
      CALL RMSERR(CITOLD(LOC1,J,2),CVP(LOC1,J),NTE,ETPDI)
      CALL RMSERR(CITOLD(LOC2,J,2),CVP(LOC2,J),NPHE,EPPDI)
  110 CONTINUE
      IFINAL=0
  120 CONTINUE
      IF(M0PR .EQ. 1 .AND. MODE .NE. 0) GO TO 320
      IF(M1PR .EQ. 1 .AND. MODE .NE. 1) GO TO 320
```

```
      DO 310 ITP=1,2
      IF(M0PR .EQ. 1) GO TO 130
      IF(ITP .EQ. 1 .AND. ETHETA(J) .EQ. 0.) GO TO 310
  130 IF(ITP .EQ. 2 .AND. EPHI(J) .EQ. 0.) GO TO 310
      PSENS=POL(ITP)
      IF(ITP .EQ. 1) GO TO 150
      DO 140 ITR=1,NUNK
      CIT(ITR,J)=CIP(ITR,J)
  140 CONTINUE
  150 CONTINUE
      PHI1=0.
      PHI2=90.
      ET1=ETTPC
      ET2=ETTDI
      EP1=EPTPC
      EP2=EPTDI
      IF(ITP .EQ. 1) GO TO 160
      ET1=ETPPC
      EP1=EPPPC
      ET2=ETPDI
      EP2=EPPDI
  160 CONTINUE
      IF(IFINAL .NE. 1) GO TO 180
      DO 170 ITR=1,NUNK
  170 CIT(ITR,J)=CITOLD(ITR,J,ITP)
  180 CONTINUE
      WRITE(JW,10009)
      WRITE(JW,10030) PSENS
      IF(IFINAL .NE. 1)      WRITE(JW,10029) MODE
      IF(IFINAL .EQ. 1) WRITE(JW,10032) MODEE
      IF(MODE .EQ. 0 .OR. IFINAL .EQ. 1) GO TO 190
      IFTC=2
      IF(ITP .EQ. 2) IFTC=1
      WRITE(JW,10031) FTC(IFTC)
  190 CONTINUE
      IF(IFINAL .NE. 1) GO TO 200
      PHIOBS=PHI1
      IF(ITP .EQ. 2) PHIOBS=PHI2
      WRITE(JW,10035) PHIOBS
  200 CONTINUE
      WRITE(JW,10010)
      DEN=2.0*PI*RHO(1)
      IF(DEN.NE.0..OR.MODE.NE.1)WRITE(JW,10011) RHO(1),Z(1),CZERO,CZERO,
     $CZERO
      IF(DEN.EQ.0..AND.MODE.EQ.1)WRITE(JW,10012) RHO(1),Z(1),CZERO,ZER
      DO 210 I=1,(NPTS-2)
      CALL MAGPHS(CIT(I,J),CMP)
      DEN=2.0*PI*RHO(I+1)
      CITD=CIT(I,J)/CMPLX(DEN,0.0)
      A=CABS(CITD)
  210 WRITE(JW,10013) RHO(I+1), Z(I+1), CIT(I,J), CMP, CITD, A
      DEN=2.0*PI*RHO(NPTS)
      IF(DEN.NE.0..OR.MODE.NE.1)WRITE(JW,10011)RHO(NPTS),Z(NPTS),
     $CZERO,CZERO,CZERO
      IF(DEN.EQ.0..AND.MODE.EQ.1)WRITE(JW,10012)RHO(NPTS),Z(NPTS),CZERO,
     $ZER
      IF(IFINAL .NE. 1) WRITE(JW,10021) ACOND,CDETRM,POWER
      IF(MODE .NE. 0 .AND. M1PR .NE. 1) WRITE(JW,10034) ET1
      WRITE(JW,10014)
      WRITE(JW,10030) PSENS
      IF(IFINAL .NE. 1)      WRITE(JW,10029) MODE
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      IF(IFINAL .EQ. 1) WRITE(JW,10032) MODEE
      IF(MODE .EQ. 0 .OR. IFINAL .EQ. 1) GO TO 220
      IFTC=1
      IF(ITP .EQ. 2) IFTC=2
      WRITE(JW,10031) FTC(IFTC)
  220 CONTINUE
      IF(IFINAL .NE. 1) GO TO 230
      PHIOBS=PHI2
      IF(ITP .EQ. 2) PHIOBS=PHI1
      WRITE(JW,10035) PHIOBS
  230 CONTINUE
      WRITE(JW,10010)
      IS=NPTS-1
      IE=NPTS*2-3
      DO 240 I=IS,IE
      IR=I-IS+1
      CALL MAGPHS(CIT(I,J),CMP)
      AMD=REAL(CMP)
      PHASE=AIMAG(CMP)
      DEN=2.0*PI*RHOPH(IR)
      CITT=CIT(I,J)*CMPLX(DEN,0.0)
      A=CABS(CITT)
  240 WRITE(JW,10013) RHOPH(IR), ZPH(IR), CITT, A, PHASE, CIT(I,J), AMD
      IF(IFINAL .NE. 1) WRITE(JW,10021) ACOND,CDETRM,POWER
      IF(MODE .NE. 0 .AND. M1PR .NE. 1) WRITE(JW,10034) EP1
      IF(IDB .EQ. 0) GO TO 310
      WRITE(JW,10015)
      WRITE(JW,10030) PSENS
      IF(IFINAL .NE. 1)      WRITE(JW,10029) MODE
      IF(IFINAL .EQ. 1) WRITE(JW,10032) MODEE
      IF(MODE .EQ. 0 .OR. IFINAL .EQ. 1) GO TO 250
      IFTC=1
      IF(ITP .EQ. 2) IFTC=2
      WRITE(JW,10031) FTC(IFTC)
  250 CONTINUE
      IF(IFINAL .NE. 1) GO TO 260
      PHIOBS=PHI2
      IF(ITP .EQ. 2) PHIOBS=PHI1
      WRITE(JW,10035) PHIOBS
  260 CONTINUE
      WRITE(JW,10010)
      DEN=2.0*PI*RHO(1)
      IF(DEN.NE.0..OR.MODE.NE.1)WRITE(JW,10011) RHO(1),Z(1),CZERO,CZERO,
     $CZERO
      IF(DEN.EQ.0..AND.MODE.EQ.1)WRITE(JW,10012) RHO(1),Z(1),CZERO,ZER
      IS=IS+NPTS-1
      IE=IE+NPTS-2
      DO 270 I=IS,IE
      IR=I-IS+2
      CALL MAGPHS(CIT(I,J),CMP)
      DEN=2.0*PI*RHO(IR)
      CITD=CIT(I,J)/CMPLX(DEN,0.0)
      A=CABS(CITD)
  270 WRITE(JW,10013) RHO(IR), Z(IR), CIT(I,J), CMP, CITD, A
      DEN =2.0*PI*RHO(NPTS)
      IF(DEN.NE.0..OR.MODE.NE.1)WRITE(JW,10011)RHO(NPTS),Z(NPTS),
     $CZERO,CZERO,CZERO
      IF(DEN.EQ.0..AND.MODE.EQ.1)WRITE(JW,10012)RHO(NPTS),Z(NPTS),CZERO,
     $ZER
      IF(IFINAL .NE. 1) WRITE(JW,10021) ACOND,CDETRM,POWER
      IF(MODE .NE. 0 .AND. M1PR .NE. 1) WRITE(JW,10034) ET2
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      WRITE(JW,10016)
      WRITE(JW,10030) PSENS
      IF(IFINAL .NE. 1)      WRITE(JW,10029) MODE
      IF(IFINAL .EQ. 1) WRITE(JW,10032) MODEE
      IF(MODE .EQ. 0 .OR. IFINAL .EQ. 1) GO TO 280
      IFTC=2
      IF(ITP .EQ. 2) IFTC=1
      WRITE(JW,10031) FTC(IFTC)
  280 CONTINUE
      IF(IFINAL .NE. 1) GO TO 290
      PHIOBS=PHI1
      IF(ITP .EQ. 2) PHIOBS=PHI2
      WRITE(JW,10035) PHIOBS
  290 CONTINUE
      WRITE(JW,10010)
      IS=IS+NPTS-2
      DO 300 I=IS,NUNK
      IR=I-IS+1
      CALL MAGPHS(CIT(I,J),CMP)
      AMD=REAL(CMP)
      PHASE=AIMAG(CMP)
      DEN=2.0*PI*RHOPH(IR)
      CITT=CIT(I,J)*CMPLX(DEN,0.0)
      A=CABS(CITT)
  300 WRITE(JW,10013) RHOPH(IR), ZPH(IR), CITT, A, PHASE, CIT(I,J), AMD
      IF(IFINAL .NE. 1) WRITE(JW,10021) ACOND,CDETRM,POWER
      IF(MODE .NE. 0 .AND. M1PR .NE. 1) WRITE(JW,10034) EP2
  310 CONTINUE
  320 CONTINUE
      IF(JM .EQ. NMODEM .AND. NMODEM .NE. 1 .AND. MODEB .EQ. 0)
     $IFINAL = IFINAL + 1
      IF(IFINAL .EQ. 1) GO TO 120
  330 CONTINUE
  340 CONTINUE
10000 FORMAT(///'0POINT'6X,'RHO'13X,'Z'11X,'DELTA-T'9X,'GAMMA'//)
10001 FORMAT(I4,1P2E15.6)
10002 FORMAT(4X,1P4E15.6)
10003 FORMAT('1'14('*')/' *'12X,'*'/' * INPUT DATA *'/' *'12X,'*'/
     $' '14('*')///)
10004 FORMAT(' MAXIMUM NUMBER OF MODES TO BE USED = 'I3//' ORDER OF GAUS
     $SIAN QUADRATURE = 'I3/)
10005 FORMAT(' FREE SPACE WAVELENGTH = '1PE/)
10006 FORMAT(' A PERFECT CONDUCTOR HAS BEEN ASSUMED.'/)
10007 FORMAT(' A DIELECTRIC BODY HAS BEEN ASSUMED:'//6X,'RELATIVE DIELEC
     $TRIC CONSTANT = '1PE//6X,'RELATIVE PERMEABILITY = '1PE/)
10008 FORMAT(' INCIDENT FIELD DATA:'//6X,'THETA = '1PE//6X,'PHI = '1PE//
     &6X,'E-THETA = '1PE//6X,'E-PHI = '1PE/)
10009 FORMAT('1'43X,'"T" COMPONENT OF ELECTRIC CURRENT')
10010 FORMAT(7X,'RHO'11X,'Z'10X,'REAL'7X,'IMAGINARY'4X,'MAGNITUDE'6X,
     $'PHASE'8X,'REAL'7X,'IMAGINARY'4X,'MAGNITUDE'/32X,'TOTAL'8X,
     $'TOTAL'8X,'TOTAL'20X,'DENSITY'6X,'DENSITY'6X,'DENSITY'/6X,
     $' ***'10X,'***'9X,'****'7X,9('*'),4X,9('*'),6X,'*****'8X,
     $'****'7X,9('*'),4X,9('*')//)
10011 FORMAT(' '1P5E13.5,'       ???   '1P3E13.5)
10012 FORMAT(' '1P5E13.5,1X,4('      ???     '))
10013 FORMAT(' '1P9E13.5)
10014 FORMAT('1'42X,'"PHI" COMPONENT OF ELECTRIC CURRENT')
10015 FORMAT('1'43X,'"T" COMPONENT OF MAGNETIC CURRENT')
10016 FORMAT('1'42X,'"PHI" COMPONENT OF MAGNETIC CURRENT')
10017 FORMAT('0'17('*')/' *'15X,'*'/' * COMPUTED DATA *'/' *'15X,'*'/1X,
     $17('*')///)
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10018 FORMAT(5X,'DIELECTRIC CONSTANT = '1PE//5X,'PERMEABILITY = '1PE//5X
     $,'WAVENUMBER = '1PE//5X,'SPEED OF LIGHT = '1PE/)
10019 FORMAT(' PARAMETERS OF THE DIELECTRIC BODY:'/)
10020 FORMAT(' OMEGA = '1PE//' FREQUENCY = '1PE/)
10021 FORMAT(/'0INV. COND. NO.:'1PE,3X,'DETERMINANT: ('
     $1P2E,') TIMES (10.)**('F6.1,')')
10022 FORMAT(' PARAMETERS OF FREE SPACE:'/)
10023 FORMAT(5X,'WAVELENGTH = '1PE/)
10024 FORMAT(' THE MAXIMUM NUMBER OF MODES WAS USED:'/' THE MAXIMUM (OV
     $ER NO. OF INC. FIELDS) "RMS" ERROR IN THE CURRENT VECTORS WAS:'
     $1PE/)
10025 FORMAT(' A DELTA-GAP SOURCE IS ASSUMED AT RHO ='1PE,' AND Z ='
     $1PE/)
10026 FORMAT(46X,'OUTPUT IN "MODE 1" FORMAT'/)
10027 FORMAT(8E)
10028 FORMAT(8I)
10029 FORMAT('0'52X,'MODE NUMBER 'I2/)
10030 FORMAT(39X,'FOR 'A5,' DIRECTED PORTION OF INCIDENT FIELD')
10031 FORMAT(49X,A3,'(PHI) COEFFICIENTS'/)
10032 FORMAT('0'47X,'SUM OF MODES 0 THROUGH 'I2/)
10033 FORMAT('0WARNING: TOO MANY INCIDENT FIELDS HAVE BEEN USED.  ONLY T
     $HE FIRST 11 HAVE BEEN PRINTED.')
10034 FORMAT('0ROOT-MEAN-SQUARE ERROR AT THIS STAGE: '1PE)
10035 FORMAT(45X,'OBSERVED AT PHI = 'F5.1,' DEGREES'/)
10036 FORMAT('1'50X,27('*')/51X,'*'25X,'*'/51X,'*  CASE NUMBER:'
     $ I3,8X,'*'/51X,'*'25X,'*'/51X,'*  EXCITATION NUMBER:'I3,2X,
     $ '*'/51X,'*'25X,'*'/51X,27('*')/)
11100 FORMAT(1P2E,10X,1P2E)
11200 FORMAT(3(' CZ('I2,',',I2,')='1P2E,1X))
      RETURN
      END
      SUBROUTINE ZMATRX(CZ,RHO,Z,RHOPH,ZPH,DELTAT,GAMMA,NUNK,M)
C***********************************************************************
C
C     SUBROUTINE "ZMATRX" FILLS THE IMPEDANCE MATRIX FOR EACH MODE.
C
C***********************************************************************
      IMPLICIT COMPLEX (C)
      COMPLEX G1R,G2R,G4R,G5R
      COMPLEX G0,G1,G2,G3,G5,G6
      COMPLEX G1M,G2M
      REAL MU1,MU2,COSG
      DIMENSION CZ(NUNK,NUNK)
      DIMENSION RHO(1),Z(1),RHOPH(1),ZPH(1),DELTAT(1),GAMMA(1)
      COMMON/PNT/NPTS,NUNKT,NUNKPH,NUNK2
      COMMON/INT/NMODEM,IDB,NGQ
      COMMON/FRQ/PI,AK1,AK2,SL1,SL2,OMEGA
      COMMON/PRM/EPS1,EPS2,MU1,MU2
      COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SING,COSG,GM
      COMMON/TPI/TWOPI,MAXP,ER,API
      COMMON/SLF/ISELF
      EXTERNAL G1,G2,G3,G5,G6,G0,G1R,G2R,G4R,G5R
      EXTERNAL G1M,G2M
      CRX(DUMMY)=CMPLX(DUMMY,0.0)
      CIX(DUMMY)=CMPLX(0.0,DUMMY)
      NPTSM1=NPTS-1
      ER=1.E-2
      NTE=NUNKT
      MAXP=10000
      NPHE=NUNKPH
      API=PI
```

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      TWOPI=2.0*PI
      IF(IDB .EQ. 0) GO TO 10
      NTE=NTE/2
      NPHE=NPHE/2
      NTM=NTE
      NPHM=NPHE
   10 MODE=M
      MODEP1=MODE+1
      MODEM1=MODE-1
      NGQ2=NGQ*2
      KTS=NTE+NPHE
      KPHS=KTS+NTM
      CH=CMPLX(0.5,0.0)
      CZERO=CMPLX(0.0,0.0)
      CJO=CMPLX(0.0,OMEGA)
      CMU1=CMPLX(MU1,0.0)
      CMU2=CMPLX(MU2,0.0)
      CEPS1=CMPLX(EPS1,0.0)
      CEPS2=CMPLX(EPS2,0.0)
      CO=CMPLX(OMEGA,0.0)
      CMP=CMPLX(FLOAT(MODEP1),0.0)
      CM=CMPLX(FLOAT(MODE),0.0)
      CMM=CMPLX(FLOAT(MODEM1),0.0)
      CM2=CM*CM
C
C-----EVALUATE ALL EXPRESSIONS IN WHICH THE FIELD POINT IS LOCATED AT
C     A POSSIBLE BEND (NON-HALF POINTS)
C
      TL=0.0
      TM=0.5
      TU=1.0
      DO 100 IF1=1,NTE
      IF3=IF1+KTS
      IF1P1=IF1+1
      RK=RHO(IF1P1)
      ZK=Z(IF1P1)
      DTK=DELTAT(IF1)
      DTKP1=DELTAT(IF1P1)
      SGK=SIN(GAMMA(IF1))
      SGKP1=SIN(GAMMA(IF1P1))
      ACGK=COS(GAMMA(IF1))
      ACGKP1=COS(GAMMA(IF1P1))
      BRCKTS=(DTKP1*SGKP1+DTK*SGK)/2.0
      BRCKTC=(DTKP1*ACGKP1+DTK*ACGK)/2.0
      DO 90 IS1=1,NPHE
      IS2=IS1+NTE
      IS3=IS1+KTS
      IS4=IS1+KPHS
      IS1M1=IS1-1
      IS3M1=IS3-1
      RIM1=RHO(IS1)
      ZIM1=Z(IS1)
      DTI=DELTAT(IS1)
      SGI=SIN(GAMMA(IS1))
      ACGI=COS(GAMMA(IS1))
      RF=RK
      ZF=ZK
      RSL=RIM1
      ZSL=ZIM1
      DEL=DTI
      SING=SGI
```

```
      COSG=ACGI
      C1B11=CIX(DTI*OMEGA*SGI*BRCKTS/2.0)
      C3B11=CIX(DTI*OMEGA*ACGI*BRCKTC)
      C1B12=CRX(-DTI*OMEGA*PI*BRCKTS)
      IF(IDB .EQ. 0) GO TO 20
      C1B13=CIX(-DTI*ACGI*BRCKTS)
      C3B13=CIX(DTI*SGI*RK*BRCKTC)
      C5B13=CIX(-DTI*SGI*BRCKTS)
      C1B14=CRX(DTI*TWOPI*BRCKTC)
      C2B14=CRX(-4.0*PI*DTI*RK*BRCKTC)
      C3B14=CRX(-DTI*TWOPI*BRCKTS)
   20 TF=TL
      ISELF=0
      IF(IS1 .EQ. IF1P1) ISELF=1
      CALL ELLPTC(TL,TM,NGQ,CANEL)
      CALL ELLPTR(TL,TM,NGQ,CANELR)
      GM=FLOAT(MODEP1)
      CALL CGQ1T(G1M,TL,TM,NGQ,CANS,CANSR)
      CA1LP=CANS+CANEL
      CA1LPR=CANSR+CANELR
      GM=FLOAT(MODEM1)
      CALL CGQ1T(G1M,TL,TM,NGQ,CANS,CANSR)
      CA1LM=CANS+CANEL
      CA1LMR=CANSR+CANELR
      IF(IS1 .EQ. 1) GO TO 30
      GM=FLOAT(MODE)
      CALL CGQ1(G1M,TL,TM,NGQ,CANS)
      CA1L=CANS+CANEL
      CZ(IF1,IS1M1)=CZ(IF1,IS1M1)+C1B11*CMU1*(CA1LP+CA1LM)
      CZ(IF1,IS1M1)=CZ(IF1,IS1M1)+C3B11*CMU1*CA1L
      IF(IDB .EQ. 0) GO TO 50
      CZ(IF3,IS3M1)=CZ(IF3,IS3M1)+C1B11*CEPS1*(CA1LP+CA1LM)
      CZ(IF3,IS3M1)=CZ(IF3,IS3M1)+C3B11*CEPS1*CA1L
   30 IF(IDB .EQ. 0) GO TO 50
      GM=FLOAT(MODEP1)
      CALL CGQ1T(G2M,TL,TM,NGQ,CANS,CANSR)
      CA2LP=CANS+CANEL
      CA2LPR=CANSR+CANELR
      GM=FLOAT(MODEM1)
      CALL CGQ1T(G2M,TL,TM,NGQ,CANS,CANSR)
      CA2LM=CANS+CANEL
      CA2LMR=CANSR+CANELR
      GM=FLOAT(MODE)
      IF(IS1 .EQ. 1) GO TO 40
      CALL CGQ1(G2M,TL,TM,NGQ,CANS)
      CA2L=CANS+CANEL
      CZ(IF1,IS1M1)=CZ(IF1,IS1M1)+C1B11*CMU2*(CA2LP+CA2LM)
      CZ(IF1,IS1M1)=CZ(IF1,IS1M1)+C3B11*CMU2*CA2L
      CZ(IF3,IS3M1)=CZ(IF3,IS3M1)+C1B11*CEPS2*(CA2LP+CA2LM)
      CZ(IF3,IS3M1)=CZ(IF3,IS3M1)+C3B11*CEPS2*CA2L
      CALL CGQ1(G0,TL,TM,NGQ,CANS0L)
      IF(ISELF .EQ. 1) CANS0L=CANS0L+CANALY(TL,TM,0,0)
      CALL CGQ1(G1,TL,TM,NGQ,CANS1L)
      CALL CGQ1(G3,TL,TM,NGQ,CANS3L)
      IF(ISELF .EQ. 1) CANS3L=CANS3L+CANALY(TL,TM,3,0)
      CB13=C1B13*CANS3L+C3B13*CANS0L+C5B13*CANS1L
      CZ(IF1,IS3M1)=CZ(IF1,IS3M1) + CB13
   40 CALL CGQ1(G4R,TL,TM,NGQ,CANS4L)
      CALL CGQ1(G5R,TL,TM,NGQ,CANS5L)
      IF(ISELF .EQ. 1) CANS5L=CANS5L+CANALY(TL,TM,5,1)
      CALL CGQ1(G2R,TL,TM,NGQ,CANS2L)
```

```
   50 TF=TU
      ISELF=0
      IF(IS1 .EQ. IF1) ISELF=1
      CALL ELLPTC(TM,TU,NGQ,CANEU)
      CALL ELLPTR(TM,TU,NGQ,CANEUR)
      GM=FLOAT(MODEP1)
      CALL CGQ1T(G1M,TM,TU,NGQ,CANS,CANSR)
      CA1UP=CANS+CANEU
      CA1UPR=CANSR+CANEUR
      GM=FLOAT(MODEM1)
      CALL CGQ1T(G1M,TM,TU,NGQ,CANS,CANSR)
      CA1UM=CANS+CANEU
      CA1UMR=CANSR+CANEUR
      IF(IS1 .EQ. NPHE) GO TO 60
      GM=FLOAT(MODE)
      CALL CGQ1(G1M,TM,TU,NGQ,CANS)
      CA1U=CANS+CANEU
      CZ(IF1,IS1)=CZ(IF1,IS1)+C1B11*CMU1*(CA1UP+CA1UM)+C3B11*CMU1*CA1U
      IF(IDB .EQ. 0) GO TO 80
      CZ(IF3,IS3)=CZ(IF3,IS3)+C1B11*CEPS1*(CA1UP+CA1UM)+C3B11*CEPS1*CA1U
   60 IF(IDB .EQ. 0) GO TO 80
      GM=FLOAT(MODEP1)
      CALL CGQ1T(G2M,TM,TU,NGQ,CANS,CANSR)
      CA2UP=CANS+CANEU
      CA2UPR=CANSR+CANEUR
      GM=FLOAT(MODEM1)
      CALL CGQ1T(G2M,TM,TU,NGQ,CANS,CANSR)
      CA2UM=CANS+CANEU
      CA2UMR=CANSR+CANEUR
      GM=FLOAT(MODE)
      IF(IS1 .EQ. NPHE) GO TO 70
      CALL CGQ1(G2M,TM,TU,NGQ,CANS)
      CA2U=CANS+CANEU
      CZ(IF1,IS1)=CZ(IF1,IS1)+C1B11*CMU2*(CA2UP+CA2UM)+C3B11*CMU2*CA2U
      CZ(IF3,IS3)=CZ(IF3,IS3)+C1B11*CEPS2*(CA2UP+CA2UM)+C3B11*CEPS2*CA2U
      CALL CGQ1(G0,TM,TU,NGQ,CANS0U)
      IF(ISELF .EQ. 1) CANS0U=CANS0U+CANALY(TM,TU,0,0)
      CALL CGQ1(G1,TM,TU,NGQ,CANS1U)
      CALL CGQ1(G3,TM,TU,NGQ,CANS3U)
      IF(ISELF .EQ. 1) CANS3U=CANS3U+CANALY(TM,TU,3,0)
      CB13=C1B13*CANS3U+C3B13*CANS0U+C5B13*CANS1U
      CZ(IF1,IS3)=CZ(IF1,IS3) + CB13
   70 CALL CGQ1(G4R,TM,TU,NGQ,CANS4U)
      CALL CGQ1(G5R,TM,TU,NGQ,CANS5U)
      IF(ISELF .EQ. 1) CANS5U=CANS5U+CANALY(TM,TU,5,1)
      CALL CGQ1(G2R,TM,TU,NGQ,CANS2U)
      CB14=C1B14*(CANS4L+CANS4U)+C2B14*(CANS5L+CANS5U)
      CB14=CB14+C3B14*(CANS2L+CANS2U)
      CZ(IF1,IS4)=CZ(IF1,IS4) + CB14
      CA2MR=CA2LMR+CA2UMR
      CA2PR=CA2LPR+CA2UPR
      CZ(IF1,IS2)=CZ(IF1,IS2)+C1B12*CMU2*(CA2PR-CA2MR)
      CZ(IF3,IS4)=CZ(IF3,IS4)+C1B12*CEPS2*(CA2PR-CA2MR)
   80 CA1MR=CA1LMR+CA1UMR
      CA1PR=CA1LPR+CA1UPR
      CZ(IF1,IS2)=CZ(IF1,IS2)+C1B12*CMU1*(CA1PR-CA1MR)
      IF(IDB .EQ. 1) CZ(IF3,IS4)=CZ(IF3,IS4)+C1B12*CEPS1*(CA1PR-CA1MR)
   90 CONTINUE
  100 CONTINUE
C
C-----COMPUTE ALL EXPRESSIONS IN WHICH THE FIELD POINT IS LOCATED AT
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```
C     A HALF POINT ON THE GENERATING CURVE.
C
      DO 270 IF1=1,NPHE
      IF2=IF1+NTE
      IF3=IF1+KTS
      IF4=IF1+KPHS
      IF1M1=IF1-1
      IF3M1=IF3-1
      RK=RHOPH(IF1)
      ZK=ZPH(IF1)
      DTK=DELTAT(IF1)
      SGK=SIN(GAMMA(IF1))
      ACGK=COS(GAMMA(IF1))
      DO 260 IS1=1,NPHE
      IS2=IS1+NTE
      IS3=IS1+KTS
      IS4=IS1+KPHS
      IS1M1=IS1-1
      IS3M1=IS3-1
      RIM1=RHO(IS1)
      ZIM1=Z(IS1)
      DTI=DELTAT(IS1)
      SGI=SIN(GAMMA(IS1))
      ACGI=COS(GAMMA(IS1))
      RF=RK
      ZF=ZK
      RSL=RIM1
      ZSL=ZIM1
      DEL=DTI
      SING=SGI
      COSG=ACGI
      C5B11=CIX(1.0/OMEGA)
      C2B12=CRX(-DTI*TWOPI/OMEGA)
      C1B22=CIX(DTK*DTI*PI*OMEGA)
      C2B22=CIX(-DTK*DTI*TWOPI/(OMEGA*RK))
      C1B21=CRX(DTK*DTI*OMEGA*SGI/2.0)
      C3B21=CRX(-DTK/(OMEGA*RK))
      IF(IDB .EQ. 0) GO TO 110
      C1B23=CRX(-2.0*DTK*DTI*RK*ACGI)
      C3B23=CRX(-DTK*DTI*ACGI)
      C5B23=CRX(DTK*DTI*SGI)
      C1B24=CIX(-DTK*TWOPI*DTI)
  110 TF=TM
      ISELF=0
      IF(IS1 .EQ. IF1) ISELF=1
      CALL ELLPTC(TL,TM,NGQ,CANEL)
      CALL ELLPTP(TL,TM,NGQ,CANELR)
      GM=FLOAT(MODEP1)
      CALL CGQ1T(G1M,TL,TM,NGQ,CANS,CANSR)
      CA1LP=CANS+CANEL
      CA1LPR=CANSR+CANELR
      GM=FLOAT(MODEM1)
      CALL CGQ1T(G1M,TL,TM,NGQ,CANS,CANSR)
      CA1LM=CANS+CANEL
      CA1LMR=CANSR+CANELR
      GM=FLOAT(MODE)
      CALL CGQ1(G1M,TL,TM,NGQ,CANS)
      CA1L=CANS+CANEL
      IF(IS1 .EQ. 1) GO TO 120
      CZ(IF2,IS1M1)=CZ(IF2,IS1M1)+C1B21*CMU1*(CA1LP-CA1LM)
      IF(IDB .EQ. 0) GO TO 140
```

```
      CZ(IF4,IS3M1)=CZ(IF4,IS3M1)+C1B21*CEPS1*(CA1LP-CA1LM)          BR- 9060
  120 IF(IDB .EQ. 0) GO TO 140                                       BR- 9070
      GM=FLOAT(MODEP1)                                               BR- 9080
      CALL CGQ1T(G2M,TL,TM,NGQ,CANS,CANSR)                           BR- 9090
      CA2LP=CANS+CANEL                                               BR- 9100
      CA2LPR=CANSR+CANELR                                            BR- 9110
      GM=FLOAT(MODEM1)                                               BR- 9120
      CALL CGQ1T(G2M,TL,TM,NGQ,CANS,CANSR)                           BR- 9130
      CA2LM=CANS+CANEL                                               BR- 9140
      CA2LMR=CANSR+CANELR                                            BR- 9150
      GM=FLOAT(MODE)                                                 BR- 9160
      CALL CGQ1(G2M,TL,TM,NGQ,CANS)                                  BR- 9170
      CA2L=CANS+CANEL                                                BR- 9180
      IF(IS1 .EQ. 1) GO TO 130                                       BR- 9190
      CZ(IF2,IS1M1)=CZ(IF2,IS1M1)+C1B21*CMU2*(CA2LP-CA2LM)           BR- 9200
      CZ(IF4,IS3M1)=CZ(IF4,IS3M1)+C1B21*CEPS2*(CA2LP-CA2LM)          BR- 9210
      CALL CGQ1(G5,TL,TM,NGQ,CANS5L)                                 BR- 9220
      IF(ISELF .EQ. 1) CANS5L=CANS5L+CANALY(TL,TM,5,0)               BR- 9230
      CALL CGQ1(G6,TL,TM,NGQ,CANS6L)                                 BR- 9240
      CALL CGQ1(G2,TL,TM,NGQ,CANS2L)                                 BR- 9250
      CB23=C1B23*CANS5L+C3B23*CANS6L+C5B23*CANS2L                    BR- 9260
      CZ(IF2,IS3M1)=CZ(IF2,IS3M1) + CB23                             BR- 9270
  130 CALL CGQ1(G1R,TL,TM,NGQ,CANS1L)                                BR- 9280
  140 CONTINUE                                                       BR- 9290
      CALL ELLPTC(TM,TU,NGQ,CANEU)                                   BR- 9300
      CALL ELLPTR(TM,TU,NGQ,CANEUR)                                  BR- 9310
      GM=FLOAT(MODEP1)                                               BR- 9320
      CALL CGQ1T(G1M,TM,TU,NGQ,CANS,CANSR)                           BR- 9330
      CA1UP=CANS+CANEU                                               BR- 9340
      CA1UPR=CANSR+CANEUR                                            BR- 9350
      GM=FLOAT(MODEM1)                                               BR- 9360
      CALL CGQ1T(G1M,TM,TU,NGQ,CANS,CANSR)                           BR- 9370
      CA1UM=CANS+CANEU                                               BR- 9380
      CA1UMR=CANSR+CANEUR                                            BR- 9390
      GM=FLOAT(MODE)                                                 BR- 9400
      CALL CGQ1(G1M,TM,TU,NGQ,CANS)                                  BR- 9410
      CA1U=CANS+CANEU                                                BR- 9420
      CA1=CA1L+CA1U                                                  BR- 9430
      CA1PR=CA1LPR+CA1UPR                                            BR- 9440
      CA1MR=CA1LMR+CA1UMR                                            BR- 9450
      IF(IS1 .EQ. NPHE) GO TO 150                                    BR- 9460
      CZ(IF2,IS1)=CZ(IF2,IS1)+C1B21*CMU1*(CA1UP-CA1UM)               BR- 9470
      IF(IDB .EQ. 0) GO TO 230                                       BR- 9480
      CZ(IF4,IS3)=CZ(IF4,IS3)+C1B21*CEPS1*(CA1UP-CA1UM)              BR- 9490
  150 IF(IDB .EQ. 0) GO TO 230                                       BR- 9500
      GM=FLOAT(MODEP1)                                               BR- 9510
      CALL CGQ1T(G2M,TM,TU,NGQ,CANS,CANSR)                           BR- 9520
      CA2UP=CANS+CANEU                                               BR- 9530
      CA2UPR=CANSR+CANEUR                                            BR- 9540
      GM=FLOAT(MODEM1)                                               BR- 9550
      CALL CGQ1T(G2M,TM,TU,NGQ,CANS,CANSR)                           BR- 9560
      CA2UM=CANS+CANEU                                               BR- 9570
      CA2UMR=CANSR+CANEUR                                            BR- 9580
      GM=FLOAT(MODE)                                                 BR- 9590
      CALL CGQ1(G2M,TM,TU,NGQ,CANS)                                  BR- 9600
      CA2U=CANS+CANEU                                                BR- 9610
      IF(IS1 .EQ. NPHE) GO TO 160                                    BR- 9620
      CZ(IF2,IS1)=CZ(IF2,IS1)+C1B21*CMU2*(CA2UP-CA2UM)               BR- 9630
      CZ(IF4,IS3)=CZ(IF4,IS3)+C1B21*CEPS2*(CA2UP-CA2UM)              BR- 9640
      CALL CGQ1(G5,TM,TU,NGQ,CANS5U)                                 BR- 9650
      IF(ISELF .EQ. 1) CANS5U=CANS5U+CANALY(TM,TU,5,0)               BR- 9660
```

```
      CALL CGQ1(G6,TM,TU,NGQ,CANS6U)
      CALL CGQ1(G2,TM,TU,NGQ,CANS2U)
      CB23=C1B23*CANS5U+C3B23*CANS6U+C5B23*CANS2U
      CZ(IF2,IS3)=CZ(IF2,IS3) + CB23
  160 CALL CGQ1(G1R,TM,TU,NGQ,CANS1U)
      CZ(IF2,IS4)=CZ(IF2,IS4)+C1B24*(CANS1L+CANS1U)
      CA2=CA2L+CA2U
      CA2PR=CA2LPR+CA2UPR
      CA2MR=CA2LMR+CA2UMR
      CA2EPS=CA2/CEPS2
      CAMU=CA1/CMU1+CA2/CMU2
      CZ(IF2,IS2)=CZ(IF2,IS2)+C1B22*CMU2*(CA2PR+CA2MR)
      CZ(IF2,IS2)=CZ(IF2,IS2)+C2B22*CA2EPS*CM2
      CB44=C1B22*(CEPS1*(CA1PR+CA1MR)+CEPS2*(CA2PR+CA2MR))
      CB44=CB44+CM2*C2B22*CAMU
      CZ(IF4,IS4)=CZ(IF4,IS4) + CB44
      CB12=C2B12*CM*CA2EPS
      CB34=C2B12*CM*CAMU
      CB21=C3B21*CM*CA2EPS
      CB43=C3B21*CM*CAMU
      CB11=C5B11*CA2EPS
      CB33=C5B11*CAMU
      IF(IF1 .EQ. 1) GO TO 170
      CZ(IF1M1,IS2)=CZ(IF1M1,IS2) + CB12
      CZ(IF3M1,IS4)=CZ(IF3M1,IS4) + CB34
  170 IF(IF1 .EQ. NPHE) GO TO 180
      CZ(IF1,IS2)=CZ(IF1,IS2) - CB12
      CZ(IF3,IS4)=CZ(IF3,IS4) - CB34
  180 IF(IS1 .EQ. 1) GO TO 200
      CZ(IF2,IS1M1)=CZ(IF2,IS1M1) - CB21
      CZ(IF4,IS3M1)=CZ(IF4,IS3M1) - CB43
      IF(IF1 .EQ. 1) GO TO 190
      CZ(IF1M1,IS1M1)=CZ(IF1M1,IS1M1) - CB11
      CZ(IF3M1,IS3M1)=CZ(IF3M1,IS3M1) - CB33
  190 IF(IF1 .EQ. NPHE) GO TO 200
      CZ(IF1,IS1M1)=CZ(IF1,IS1M1) + CB11
      CZ(IF3,IS3M1)=CZ(IF3,IS3M1) + CB33
  200 IF(IS1 .EQ. NPHE) GO TO 220
      CZ(IF2,IS1)=CZ(IF2,IS1) + CB21
      CZ(IF4,IS3)=CZ(IF4,IS3) + CB43
      IF(IF1 .EQ. 1) GO TO 210
      CZ(IF1M1,IS1)=CZ(IF1M1,IS1) + CB11
      CZ(IF3M1,IS3)=CZ(IF3M1,IS3) + CB33
  210 IF(IF1 .EQ. NPHE) GO TO 220
      CZ(IF1,IS1)=CZ(IF1,IS1) - CB11
      CZ(IF3,IS3)=CZ(IF3,IS3) - CB33
  220 CONTINUE
  230 CONTINUE
      CA1EPS=CA1/CEPS1
      CB21=C3B21*CM*CA1EPS
      CB12=C2B12*CM*CA1EPS
      CB11=C5B11*CA1EPS
      CZ(IF2,IS2)=CZ(IF2,IS2)+C1B22*CMU1*(CA1PR+CA1MR)
      CZ(IF2,IS2)=CZ(IF2,IS2)+C2B22*CA1EPS*CM2
      IF(IF1 .NE. 1) CZ(IF1M1,IS2)=CZ(IF1M1,IS2) + CB12
      IF(IF1 .NE. NPHE) CZ(IF1,IS2)=CZ(IF1,IS2) - CB12
      IF(IS1 .EQ. 1) GO TO 240
      CZ(IF2,IS1M1)=CZ(IF2,IS1M1) - CB21
      IF(IF1 .NE. 1) CZ(IF1M1,IS1M1)=CZ(IF1M1,IS1M1) - CB11
      IF(IF1 .NE. NPHE)   CZ(IF1,IS1M1)=CZ(IF1,IS1M1) + CB11
  240 IF(IS1 .EQ. NPHE) GO TO 250
```

```
      CZ(IF2,IS1)=CZ(IF2,IS1) + CB21
      IF(IF1 .NE. 1) CZ(IF1M1,IS1)=CZ(IF1M1,IS1) + CB11
      IF(IF1 .NE. NPHE) CZ(IF1,IS1)=CZ(IF1,IS1) - CB11
  250 CONTINUE
  260 CONTINUE
  270 CONTINUE
C
C-----NOW WE HAVE:
C
C          (BETA-31) = -(BETA-13)
C          (BETA-32) = -(BETA-14)
C          (BETA-41) = -(BETA-23)
C          (BETA-42) = -(BETA-24)
C
      IF(IDB .EQ. 0) GO TO 290
      NE=NTE+NPHE
      NE1=NE+1
      DO 280 IF=1,NE
      IFP=IF+NE
      DO 280 IS=NE1,NUNK
      ISP=IS-NE
      CZ(IFP,ISP)= -CZ(IF,IS)
  280 CONTINUE
  290 CONTINUE
      RETURN
      END
      SUBROUTINE CVFILL(CVTHC,CVPHC,RHO,Z,RHOPH,ZPH,DELTAT,GAMMA,
     #                  THETA,PHI,ETHETA,EPHI,ANTFD,NUNK,NFLDS,MODE)
C***********************************************************************
C
C     SUBROUTINE CVFILL COMPUTES THE FORCING FUNCTION VECTOR (OR
C     MATRIX FOR MULTIPLE EXCITATIONS).
C
C***********************************************************************
      IMPLICIT COMPLEX (C)
      DOUBLE PRECISION BARG1,BJ(50),BY(2)
      REAL COSTH,CGK,CGKP1,MU
      DIMENSION CVTHC(NUNK,NFLDS),CVPHC(NUNK,NFLDS)
      DIMENSION RHO(1),Z(1),RHOPH(1),ZPH(1),GAMMA(1),DELTAT(1)
      DIMENSION THETA(1),PHI(1),ETHETA(1),EPHI(1),ANTFD(1)
      COMMON/PNT/NPTS,NUNKT,NUNKPH,NUNK2
      COMMON/INT/NMODEM,IDB,NGQ
      COMMON/FRQ/PI,AK1,AK2,SL1,SL2,OMEGA
      CALL ZERO(CVTHC,NUNK*NFLDS)
      CALL ZERO(CVPHC,NUNK*NFLDS)
      MODEP1=MODE+1
      MODEM1=MODE-1
      NTE=NUNKT
      NPHE=NUNKPH
      NTM=0
      NPHM=0
      IF(IDB .EQ. 0) GO TO 10
      NTE=NTE/2
      NPHE=NPHE/2
      NTM=NTE
      NPHM=NPHE
   10 CONTINUE
      TPI=2.0*PI
      FPI=4.0*PI
      JMP1=MODEP1+1
      JM=MODEP1
```

```
      JMM1=MODE
      MU=FPI*1.E-7
      ETA=OMEGA*MU/AK1
      CJ=CMPLX(0.0,1.0)
C
C----- EVALUATE FIELDS AT NON-HALF POINTS
C
      DO 110 IF1=1,NTE
      IF3=IF1+NTE+NPHE
      IF1P1=IF1+1
      RF=RHO(IF1P1)
      ZF=Z(IF1P1)
      DTK=DELTAT(IF1)
      DTKP1=DELTAT(IF1P1)
      SGK=SIN(GAMMA(IF1))
      SGKP1=SIN(GAMMA(IF1P1))
      CGK=COS(GAMMA(IF1))
      CGKP1=COS(GAMMA(IF1P1))
      BRCKTS=(DTKP1*SGKP1+DTK*SGK)/2.0
      BRCKTC=(DTKP1*CGKP1+DTK*CGK)/2.0
      DO 100 I=1,NFLDS
      COSTH=COS(THETA(I)*PI/180.0)
      SINTH=SIN(THETA(I)*PI/180.0)
      BARG=AK1*RF*SINTH
      BARG1=DBLE(BARG)
      IF(BARG) 20, 20, 40
   20 DO 30 K=2,JMP1
   30 BJ(K)=0.0
      BJ(1)=1.0
      GO TO 50
   40 CALL BESSEL(BARG1,MODEP1,BJ,BY,1)
   50 BET1=TPI*COSTH*BRCKTS*ETHETA(I)
      BET2=-FPI*SINTH*BRCKTC*ETHETA(I)
      BET3=TPI*BRCKTS*EPHI(I)
      BHT1=TPI*BRCKTS*COSTH*EPHI(I)/ETA
      BHT2=-FPI*BRCKTC*SINTH*EPHI(I)/ETA
      BHT3=-TPI*BRCKTS*ETHETA(I)/ETA
      CJMP1=CMPLX(SNGL(BJ(JMP1)),0.0)
      CJM=CMPLX(SNGL(BJ(JM)),0.0)
      IF(JMM1) 60, 60, 70
   60 CJMM1=-CJMP1
      GO TO 80
   70 CJMM1=CMPLX(SNGL(BJ(JMM1)),0.0)
   80 CONTINUE
      CJPMM2=CJ**(MODEM1-1)
      CJPMM1=CJ*CJPMM2
      CJPM=CJ*CJPMM1
      CJPMP1=CJ*CJPM
      EARG=AK1*ZF*COSTH-PHI(I)*FLOAT(MODE)*PI/180.0
      CARG=CMPLX(0.0,EARG)
      CE=CEXP(CARG)
      CV=(CJMM1*CJPMM1+CJPMP1*CJMP1)*CMPLX(BET1,0.0)
      CV=CV+CJPM*CJM*CMPLX(BET2,0.0)
      CVTHC(IF1,I)=CV*CE
      CV=(CJPMM2*CJMM1-CJPM*CJMP1)*CMPLX(BET3,0.0)
      CVPHC(IF1,I)=CV*CE
      IF(IDB) 90, 100, 90
   90 CV=(CJPMM2*CJMM1-CJPM*CJMP1)*CMPLX(BHT3,0.0)
      CVTHC(IF3,I)=CV*CE
      CV=(CJPMM1*CJMM1+CJPMP1*CJMP1)*CMPLX(BHT1,0.0)
      CV=CV+CJPM*CJM*CMPLX(BHT2,0.0)
```

```
      CVPHC(IF3,I)=CV*CE
  100 CONTINUE
  110 CONTINUE
C
C----- EVALUATE FIELDS AT HALF POINTS
C
      DO 200 IF1=1,NPHE
      IF2=IF1+NTE
      IF4=IF2+NPHE+NTM
      RF=RHOPH(IF1)
      ZF=ZPH(IF1)
      DTK=DELTAT(IF1)
      DO 190 I=1,NFLDS
      COSTH=COS(THETA(I)*PI/180.0)
      SINTH=SIN(THETA(I)*PI/180.0)
      BARG=AK1*RF*SINTH
      BARG1=DBLE(BARG)
      IF(BARG) 120, 120, 140
  120 DO 130 K=2,JMP1
  130 BJ(K)=0.0
      BJ(1)=1.0
      GO TO 150
  140 CALL BESSEL(BARG1,MODEP1,BJ,BY,1)
  150 BEP1=-DTK*TPI*COSTH*ETHETA(I)
      BEP2=DTK*TPI*EPHI(I)
      BHP1=-DTK*TPI*COSTH*EPHI(I)/ETA
      BHP2=-DTK*TPI*ETHETA(I)/ETA
      CJMP1=CMPLX(SNGL(BJ(JMP1)),0.0)
      IF(JMM1) 160, 160, 170
  160 CJMM1=-CJMP1
      GO TO 180
  170 CJMM1=CMPLX(SNGL(BJ(JMM1)),0.0)
  180 CONTINUE
      CJPMM2=CJ**(MODEM1-1)
      CJPMM1=CJ*CJPMM2
      CJPM=CJ*CJPMM1
      CJPMP1=CJPM*CJ
      EARG=AK1*ZF*COSTH-PHI(I)*FLOAT(MODE)*PI/180.
      CARG=CMPLX(0.0,EARG)
      CE=CEXP(CARG)
      CV=(CJPMM2*CJMM1-CJPM*CJMP1)*CMPLX(BEP1,0.0)
      CVTHC(IF2,I)=CV*CE
      CV=(CJPMM1*CJMM1+CJPMP1*CJMP1)*CMPLX(BEP2,0.0)
      CVPHC(IF2,I)=CV*CE
      IF(IDB .EQ. 0) GO TO 190
      CV=(CJPMM2*CJMM1-CJPM*CJMP1)*CMPLX(BHP1,0.0)
      CVPHC(IF4,I)=CV*CE
      CV=(CJPMM1*CJMM1+CJPMP1*CJMP1)*CMPLX(BHP2,0.0)
      CVTHC(IF4,I)=CV*CE
  190 CONTINUE
  200 CONTINUE
C
C----- INCLUDE VOLTAGE SOURCE IF PRESENT
C
      IF(MODE .NE. 0) GO TO 220
      DO 210 I=1,NFLDS
      IANTFD=IFIX(ANTFD(I))
      IF(IANTFD .LE. 0) GO TO 210
      CVTHC(IANTFD,I)=CVTHC(IANTFD,I)+FPI
  210 CONTINUE
  220 CONTINUE
```

```
      RETURN
      END
      SUBROUTINE SUMOUT(CI,NUNK,NFLDS,MODE,ITHETA)
C***********************************************************************
C
C     SUBROUTINE "SUMOUT" TRANSFORMS THE EXPONENTIAL COMPONENT CURRENT
C     COEFFICIENTS TO SINE AND COSINE COMPONENT CURRENT COEFFICIENTS.
C
C***********************************************************************
      IMPLICIT COMPLEX (C)
      DIMENSION CI(NUNK,NFLDS)
      COMMON/INT/NMODEM,IDB,NGQ
      COMMON/PNT/NPTS,NUNKT,NUNKPH,NUNK2
      IF(MODE .EQ. 0) RETURN
      NTE=NUNKT
      NPHE=NUNKPH
      NTM=0
      NPHM=0
      IF(IDB .EQ. 0) GO TO 10
      NTE=NTE/2
      NPHE=NPHE/2
      NTM=NTE
      NPHM=NPHE
   10 CONTINUE
      CF1=CMPLX(2.0,0.0)
      CF2=CMPLX(0.0,2.0)
      IF(ITHETA .EQ. 1) GO TO 20
      CF=CF1
      CF1=CF2
      CF2=CF
   20 CONTINUE
      DO 60 J=1,NFLDS
      DO 50 I=1,NUNK
      IF(I .GT. NTE) GO TO 40
   30 CI(I,J)=CI(I,J)*CF1
      GO TO 50
   40 IF(I .GT. NTE+NPHE+NTM) GO TO 30
      CI(I,J)=CI(I,J)*CF2
   50 CONTINUE
   60 CONTINUE
      RETURN
      END
      SUBROUTINE RMSERR(CIT,CITOLD,NUNK,RMSMXE)
C***********************************************************************
C
C     SUBROUTINE "RMSERR" COMPUTES THE ROOT-MEAN-SQUARE RELATIVE
C     ERROR BETWEEN THE PRESENT CURRENT SOLUTION AND THE PREVIOUS
C     SOLUTION.
C
C***********************************************************************
      IMPLICIT COMPLEX (C)
      DIMENSION CIT(NUNK), CITOLD(NUNK)
      SQERR=0.0
      DO 10 I=1,NUNK
      DEN=CABS(CIT(I))
      ANUM=CABS(CIT(I)-CITOLD(I))
      IF(DEN .LT. 1.E-9) DEN=CABS(CITOLD(I))
      IF(DEN .LT. 1.E-9) DEN=1.0
      RELERR=ANUM/DEN
      SQERR=SQERR+RELERR**2
   10 CONTINUE
```

```
      SQMERR=SQERR/FLOAT(NUNK)
      RMSMXE=SQRT(SQMERR)
      RETURN
      END
      SUBROUTINE ZERO(C,N)
C
C----- INITIALIZATION ROUTINE
C
      COMPLEX C(N)
      C0=CMPLX(0.0,0.0)
      DO 10 J=1,N
   10 C(J)=C0
      RETURN
      END
      SUBROUTINE MAGPHS(CRI,CMP)
C
C----- ROUTINE TO CALCULATE MAGNITUDE AND PHASE OF A COMPLEX NUMBER
C       RESULT IS STORED IN A COMPLEX NUMBER AS WELL
C
      IMPLICIT COMPLEX (C)
      REAL I,M
      COMMON/FRQ/PI,A,B,D,E,F
      R=REAL(CRI)
      I=AIMAG(CRI)
      M=CABS(CRI)
      P=ATAN2(I,R)*180.0/PI
      CMP=CMPLX(M,P)
      RETURN
      END
      SUBROUTINE NORMAL(CI,ETHETA,EPHI,NUNK,NFLDS)
C*************************************************************************
C
C        SUBROUTINE NORMAL NORMALIZES THE ELECTRIC CURRENT BY THE
C        INCIDENT H-FIELD AND THE MAGNETIC CURRENT BY THE INCIDENT
C        E-FIELD.
C
C*************************************************************************
      IMPLICIT COMPLEX (C)
      DIMENSION CI(NUNK,NFLDS)
      DIMENSION ETHETA(1),EPHI(1)
      COMMON/FRQ/PI,AK1,AK2,SL1,SL2,OMEGA
      COMMON/INT/NMODEM,IDB,NGQ
      NUNKE=NUNK
      IF(IDB .EQ. 1) NUNKE=NUNK/2
      NUNKE1=NUNKE+1
      DO 30 K=1,NFLDS
      ENORM=SQRT(ETHETA(K)**2+EPHI(K)**2)
      HNORM=ENORM*SL1*8.85418533E-12
      CEN=CMPLX(ENORM,0.0)
      CHN=CMPLX(HNORM,0.0)
      DO 10 I=1,NUNKE
   10 CI(I,K)=CI(I,K)/CHN
      IF(IDB .EQ. 0) GO TO 30
      DO 20 I=NUNKE1,NUNK
   20 CI(I,K)=CI(I,K)/CEN
   30 CONTINUE
      RETURN
      END
      COMPLEX FUNCTION G1M(TP)
      IMPLICIT COMPLEX (C)
      COMMON/TPI/TWOPI,MAXP,ER,PI
```

```
      COMMON/ARC/T                                                       BR-13330
      EXTERNAL CG1M                                                      BR-13340
      T=TP                                                               BR-13350
      CALL TRPADP(CG1M,0.0,PI,ER,MAXP,IER,CANS)                          BR-13360
      G1M=CANS/CMPLX(TWOPI,0.0)                                          BR-13370
      RETURN                                                             BR-13380
      END                                                                BR-13390
      COMPLEX FUNCTION CG1M(PHIP)                                        BR-13400
      IMPLICIT COMPLEX (C)                                               BR-13410
      REAL COSG                                                          BR-13420
      COMMON/ARC/TP                                                      BR-13430
      COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SING,COSG,GM                       BR-13440
      COMMON/FRQ/PI,AK1,AK2,SL1,SL2,OMEGA                                BR-13450
      RS=RSL+TP*DEL*SING                                                 BR-13460
      ZS=ZSL+TP*DEL*COSG                                                 BR-13470
      R=SQRT(RF*RF+RS*RS+(ZF-ZS)**2-2.0*RF*RS*COS(PHIP))                 BR-13480
      CARG=CMPLX(0.0,-AK1*R)                                             BR-13490
      CR=CMPLX(R,0.0)                                                    BR-13500
      ACMP=COS(GM*PHIP)                                                  BR-13510
      CG1M=(CMPLX(ACMP,0.0)*CEXP(CARG)-CMPLX(1.0,0.0))/CR                BR-13520
      RETURN                                                             BR-13530
      END                                                                BR-13540
      COMPLEX FUNCTION G2M(TP)                                           BR-13550
      IMPLICIT COMPLEX (C)                                               BR-13560
      COMMON/TPI/TWOPI,MAXP,ER,PI                                        BR-13570
      COMMON/ARC/T                                                       BR-13580
      EXTERNAL CG2M                                                      BR-13590
      T=TP                                                               BR-13600
      CALL TRPADP(CG2M,0.0,PI,ER,MAXP,IER,CANS)                          BR-13610
      G2M=CANS/CMPLX(TWOPI,0.0)                                          BR-13620
      RETURN                                                             BR-13630
      END                                                                BR-13640
      COMPLEX FUNCTION CG2M(PHIP)                                        BR-13650
      IMPLICIT COMPLEX (C)                                               BR-13660
      REAL COSG                                                          BR-13670
      COMMON/ARC/TP                                                      BR-13680
      COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SING,COSG,GM                       BR-13690
      COMMON/FRQ/PI,AK1,AK2,SL1,SL2,OMEGA                                BR-13700
      RS=RSL+TP*DEL*SING                                                 BR-13710
      ZS=ZSL+TP*DEL*COSG                                                 BR-13720
      R=SQRT(RF*RF+RS*RS+(ZF-ZS)**2-2.0*RF*RS*COS(PHIP))                 BR-13730
      CARG=CMPLX(0.0,-AK2*R)                                             BR-13740
      CR=CMPLX(R,0.0)                                                    BR-13750
      ACMP=COS(GM*PHIP)                                                  BR-13760
      CG2M=(CMPLX(ACMP,0.0)*CEXP(CARG)-CMPLX(1.0,0.0))/CR                BR-13770
      RETURN                                                             BR-13780
      END                                                                BR-13790
      SUBROUTINE ELLPTC(TL,TU,NGQ,CANS)                                  BR-13800
      IMPLICIT COMPLEX (C)                                               BR-13810
      REAL COSG                                                          BR-13820
      COMMON/TPI/TWOPI,MAXP,ER,PI                                        BR-13830
      COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SING,COSG,GM                       BR-13840
      COMMON/SLF/ISELF                                                   BR-13850
      EXTERNAL CGME                                                      BR-13860
      CALL CGQ1(CGME,TL,TU,NGQ,CANS1)                                    BR-13870
      CANS=CANS1*CMPLX(2.0/PI,0.0)                                       BR-13880
      IF(ISELF .EQ. 0) RETURN                                            BR-13890
      TLLN=0.0                                                           BR-13900
      TULN=0.0                                                           BR-13910
      IF(TF .GT. TL) TLLN=(TF-TL)*ALOG(TF-TL)                            BR-13920
      IF(TF .LT. TU) TULN=(TU-TF)*ALOG(TU-TF)                            BR-13930
```

```
      ANS=((TU-TL)*(1.0-ALOG(DEL))-TLLN-TULN)/RF/PI
      CANS=CANS+CMPLX(ANS,0.0)
      RETURN
      END
      COMPLEX FUNCTION CGME(TP)
      IMPLICIT COMPLEX (C)
      REAL COSG
      COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SING,COSG,GM
      COMMON/SLF/ISELF
      RS=RSL+TP*DEL*SING
      ZS=ZSL+TP*DEL*COSG
      ZFMZS2=(ZF-ZS)**2
      RFMRS2=(RF-RS)**2
      RN=RFMRS2+ZFMZS2
      RD=(RF+RS)**2+ZFMZS2
      R2=SQRT(RD)
      R3=SQRT(RN)
      AM1=RN/RD
      ANS=ELIC1K(AM1)
      ANS=ANS/R2
      IF(ISELF .EQ. 0) GO TO 10
      ANS=ANS+ALOG(R3)/2.0/RF
   10 CGME=CMPLX(ANS,0.0)
      RETURN
      END
      SUBROUTINE ELLPTR(TL,TU,NGQ,CANS)
      IMPLICIT COMPLEX (C)
      REAL COSG
      COMMON/TPI/TWOPI,MAXP,ER,PI
      COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SING,COSG,GM
      COMMON/SLF/ISELF
      EXTERNAL CGMER
      CALL CGQ1(CGMER,TL,TU,NGQ,CANS1)
      CANS=CANS1*CMPLX(2.0/PI,0.0)
      IF(ISELF .EQ. 0) RETURN
      TLLN=0.0
      TULN=0.0
      IF(TF .GT. TL) TLLN=(TF-TL)*ALOG(TF-TL)
      IF(TF .LT. TU) TULN=(TU-TF)*ALOG(TU-TF)
      ANS=((TU-TL)*(1.0-ALOG(DEL))-TLLN-TULN)/PI
      CANS=CANS+CMPLX(ANS,0.0)
      RETURN
      END
      COMPLEX FUNCTION CGMER(TP)
      IMPLICIT COMPLEX (C)
      REAL COSG
      COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SING,COSG,GM
      COMMON/SLF/ISELF
      RS=RSL+TP*DEL*SING
      ZS=ZSL+TP*DEL*COSG
      ZFMZS2=(ZF-ZS)**2
      RFMRS2=(RF-RS)**2
      RN=RFMRS2+ZFMZS2
      RD=(RF+RS)**2+ZFMZS2
      R2=SQRT(RD)
      R3=SQRT(RN)
      AM1=RN/RD
      ANS=ELIC1K(AM1)
      ANS=ANS*RS/R2
      IF(ISELF .EQ. 0) GO TO 10
      ANS=ANS+ALOG(R3)/2.0
```

```
   10 CGMER=CMPLX(ANS,0.0)
      RETURN
      END
      COMPLEX FUNCTION CGPOR(DUMMY)
C
C------ FUNCTION:  (1/(R0))*(D(G1+G2)/D(R0))
C
      IMPLICIT COMPLEX (C)
      REAL COSGS
      COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SINGS,COSGS,GM
      COMMON/FRQ/PI,AK1,AK2,SL1,SL2,OMEGA
      COMMON/ARC/TP
      COMMON/ANG/PHIP
      RS=RSL+TP*DEL*SINGS
      ZS=ZSL+TP*DEL*COSGS
      R=SQRT(RF*RF+RS*RS+(ZF-ZS)**2-2.0*RF*RS*COS(PHIP))
      RK1=AK1*R
      RK2=AK2*R
      CARG1=CMPLX(0.0,-RK1)
      CARG2=CMPLX(0.0,-RK2)
      C1PJK1=CMPLX(1.0,RK1)
      C1PJK2=CMPLX(1.0,RK2)
      CB=C1PJK1*CEXP(CARG1)+C1PJK2*CEXP(CARG2)
      CGPOR=-CB/CMPLX(R**3,0.0)
      RETURN
      END
      COMPLEX FUNCTION G0(TP)
      IMPLICIT COMPLEX (C)
      COMMON/TPI/TWOPI,MAXP,ER,PI
      COMMON/ARC/T
      EXTERNAL CG0
      T=TP
      CALL TRPADP(CG0,0.0,PI,ER,MAXP,IER,CANS)
      G0=CANS/CMPLX(TWOPI,0.0)
      RETURN
      END
      COMPLEX FUNCTION G1(TP)
      IMPLICIT COMPLEX (C)
      COMMON/ISWTCH/ISWR
      COMMON/TPI/TWOPI,MAXP,ER,PI
      COMMON/ARC/T
      EXTERNAL CG1
      ISWR=0
      T=TP
      CALL TRPADP(CG1,0.0,PI,ER,MAXP,IER,CANS)
      G1=CANS/CMPLX(TWOPI,0.0)
      RETURN
      END
      COMPLEX FUNCTION G1R(TP)
      IMPLICIT COMPLEX (C)
      COMMON/ISWTCH/ISWR
      COMMON/TPI/TWOPI,MAXP,ER,PI
      COMMON/ARC/T
      EXTERNAL CG1
      ISWR=1
      T=TP
      CALL TRPADP(CG1,0.0,PI,ER,MAXP,IER,CANS)
      G1R=CANS/CMPLX(TWOPI,0.0)
      RETURN
      END
      COMPLEX FUNCTION G2(TP)
```

```
      IMPLICIT COMPLEX (C)
      COMMON/TPI/TWOPI,MAXP,ER,PI
      COMMON/ARC/T
      COMMON/ISWTCH/ISWR
      EXTERNAL CG2
      T=TP
      ISWR=0
      CALL TRPADP(CG2,0.0,PI,ER,MAXP,IER,CANS)
      G2=CANS/CMPLX(TWOPI,0.0)
      RETURN
      END
      COMPLEX FUNCTION C2R(TP)
      IMPLICIT COMPLEX (C)
      COMMON/TPI/TWOPI,MAXP,ER,PI
      COMMON/ARC/T
      COMMON/ISWTCH/ISWR
      EXTERNAL CG2
      T=TP
      ISWR=1
      CALL TRPADP(CG2,0.0,PI,ER,MAXP,IER,CANS)
      G2R=CANS/CMPLX(TWOPI,0.0)
      RETURN
      END
      COMPLEX FUNCTION G3(TP)
      IMPLICIT COMPLEX (C)
      COMMON/TPI/TWOPI,MAXP,ER,PI
      COMMON/ARC/T
      EXTERNAL CG3
      T=TP
      CALL TRPADP(CG3,0.0,PI,ER,MAXP,IER,CANS)
      G3=CANS/CMPLX(TWOPI,0.0)
      RETURN
      END
      COMPLEX FUNCTION G4R(TP)
      IMPLICIT COMPLEX (C)
      COMMON/TPI/TWOPI,MAXP,ER,PI
      COMMON/ARC/T
      COMMON/ISWTCH/ISWR
      EXTERNAL CG4
      T=TP
      ISWR=1
      CALL TRPADP(CG4,0.0,PI,ER,MAXP,IER,CANS)
      G4R=CANS/CMPLX(TWOPI,0.0)
      RETURN
      END
      COMPLEX FUNCTION G5(TP)
      IMPLICIT COMPLEX (C)
      COMMON/TPI/TWOPI,MAXP,ER,PI
      COMMON/ARC/T
      COMMON/ISWTCH/ISWR
      EXTERNAL CG5
      T=TP
      ISWR=0
      CALL TRPADP(CG5,0.0,PI,ER,MAXP,IER,CANS)
      G5=CANS/CMPLX(TWOPI,0.0)
      RETURN
      END
      COMPLEX FUNCTION G5R(TP)
      IMPLICIT COMPLEX (C)
      COMMON/TPI/TWOPI,MAXP,ER,PI
      COMMON/ARC/T
```

```
      COMMON/ISWTCH/ISWR
      EXTERNAL CG5
      T=TP
      ISWR=1
      CALL TRPADP(CG5,0.0,PI,ER,MAXP,IER,CANS)
      G5R=CANS/CMPLX(TWOPI,0.0)
      RETURN
      END
      COMPLEX FUNCTION G6(TP)
      IMPLICIT COMPLEX (C)
      COMMON/TPI/TWOPI,MAXP,ER,PI
      COMMON/ARC/T
      EXTERNAL CG6
      T=TP
      CALL TRPADP(CG6,0.0,PI,ER,MAXP,IER,CANS)
      G6=CANS/CMPLX(TWOPI,0.0)
      RETURN
      END
      COMPLEX FUNCTION CG0(PHIP)
      IMPLICIT COMPLEX (C)
      REAL COSGS
      COMMON/ANG/PHI
      COMMON/ARC/TP
      COMMON/SLF/ISELF
      COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SINGS,COSGS,GM
      COMMON/FRQ/PI,AK1,AK2,SL1,SL2,OMEGA
      PHI=PHIP
      SMPSP=SIN(GM*PHIP)*SIN(PHIP)
      CINTG=CMPLX(SMPSP,0.0)*CGPOR(XXXXXX)
      IF(ISELF .EQ. 0) GO TO 10
      ANUM=2.0*GM*PHIP*PHIP
      DEN=(((TF-TP)*DEL)**2+(RF*PHIP)**2)**1.5
      CINTG=CINTG+CMPLX(ANUM/DEN,0.0)
   10 CG0=CINTG
      RETURN
      END
      COMPLEX FUNCTION CG1(PHIP)
      IMPLICIT COMPLEX (C)
      REAL COSGS
      COMMON/ISWTCH/ISWR
      COMMON/ANG/PHI
      COMMON/ARC/TP
      COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SINGS,COSGS,GM
      COMMON/FRQ/PI,AK1,AK2,SL1,SL2,OMEGA
      PHI=PHIP
      ZS=ZSL+TP*DEL*COSGS
      SMPSP=SIN(GM*PHIP)*SIN(PHIP)
      IF(ISWR .EQ. 1) SMPSP=SMPSP*(RSL+TP*DEL*SINGS)
      CG1=CMPLX((ZF-ZS)*SMPSP,0.0)*CGPOR(XXXXXX)
      RETURN
      END
      COMPLEX FUNCTION CG2(PHIP)
      IMPLICIT COMPLEX (C)
      REAL COSGS,CMPCP
      COMMON/ANG/PHI
      COMMON/ISWTCH/ISWR
      COMMON/ARC/TP
      COMMON/SLF/ISELF
      COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SINGS,COSGS,GM
      COMMON/FRQ/PI,AK1,AK2,SL1,SL2,OMEGA
      PHI=PHIP
```

```
      ZS=ZSL+TP*DEL*COSGS
      CMPCP=COS(GM*PHIP)*COS(PHIP)
      IF(ISWR .EQ. 1) CMPCP=CMPCP*(RSL+TP*DEL*SINGS)
      CINTG=CMPLX((ZF-ZS)*CMPCP,0.0)*CGPOR(XXXXXX)
      IF(ISELF .EQ. 0) GO TO 10
      ANUM=2.0*(TF-TP)*DEL*COSGS
      IF(ISWR .EQ. 1) ANUM=ANUM*RF
      DEN=(((TF-TP)*DEL)**2+(RF*PHIP)**2)**1.5
      CINTG=CINTG+CMPLX(ANUM/DEN,0.0)
   10 CG2=CINTG
      RETURN
      END
      COMPLEX FUNCTION CG3(PHIP)
      IMPLICIT COMPLEX (C)
      REAL COSGS
      COMMON/ANG/PHI
      COMMON/ARC/TP
      COMMON/SLF/ISELF
      COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SINGS,COSGS,GM
      COMMON/FRQ/PI,AK1,AK2,SL1,SL2,OMEGA
      PHI=PHIP
      RS=RSL+TP*DEL*SINGS
      SMPSP=SIN(GM*PHIP)*SIN(PHIP)
      CINTG=CMPLX(RS*SMPSP,0.0)*CGPOR(XXXXXX)
      IF(ISELF .EQ. 0) GO TO 10
      ANUM=2.0*RF*GM*PHIP*PHIP
      DEN=(((TF-TP)*DEL)**2+(RF*PHIP)**2)**1.5
      CINTG=CINTG+CMPLX(ANUM/DEN,0.0)
   10 CG3=CINTG
      RETURN
      END
      COMPLEX FUNCTION CG4(PHIP)
      IMPLICIT COMPLEX (C)
      REAL COSGS,CMP
      COMMON/ANG/PHI
      COMMON/ISWTCH/ISWR
      COMMON/ARC/TP
      COMMON/SLF/ISELF
      COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SINGS,COSGS,GM
      COMMON/FRQ/PI,AK1,AK2,SL1,SL2,OMEGA
      PHI=PHIP
      RS=RSL+TP*DEL*SINGS
      CMP=COS(GM*PHIP)
      IF(ISWR .EQ. 1) CMP=CMP*RS
      CINTG=CMPLX((RF-RS)*CMP,0.0)*CGPOR(XXXXXX)
      IF(ISELF .EQ. 0) GO TO 10
      ANUM=2.0*(TF-TP)*DEL*SINGS
      IF(ISWR .EQ. 1) ANUM=ANUM*RF
      DEN=(((TF-TP)*DEL)**2+(RF*PHIP)**2)**1.5
      CINTG=CINTG+CMPLX(ANUM/DEN,0.0)
   10 CG4=CINTG
      RETURN
      END
      COMPLEX FUNCTION CG5(PHIP)
      IMPLICIT COMPLEX (C)
      REAL COSGS
      COMMON/ISWTCH/ISWR
      COMMON/ANG/PHI
      COMMON/ARC/TP
      COMMON/SLF/ISELF
      COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SINGS,COSGS,GM
```

```
      COMMON/FRQ/PI,AK1,AK2,SL1,SL2,OMEGA
      PHI=PHIP
      S2PCMP=COS(GM*PHIP)*(SIN(PHIP/2.0)**2)
      IF(ISWR .EQ. 1) S2PCMP=S2PCMP*(RSL+TP*DEL*SINGS)
      CINTG=CMPLX(S2PCMP,0.0)*CGPOR(XXXXXX)
      IF(ISELF .EQ. 0) GO TO 10
      ANUM=PHIP*PHIP/2.0
      IF(ISWR .EQ. 1) ANUM=ANUM*RF
      DEN=(((TF-TP)*DEL)**2+(RF*PHIP)**2)**1.5
      CINTG=CINTG+CMPLX(ANUM/DEN,0.0)
   10 CG5=CINTG
      RETURN
      END
      COMPLEX FUNCTION CG6(PHIP)
      IMPLICIT COMPLEX (C)
      REAL COSGS,CMPCP
      COMMON/ANG/PHI
      COMMON/ARC/TP
      COMMON/SLF/ISELF
      COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SINGS,COSGS,GM
      COMMON/FRQ/PI,AK1,AK2,SL1,SL2,OMEGA
      PHI=PHIP
      RS=RSL+TP*DEL*SINGS
      CMPCP=COS(GM*PHIP)*COS(PHIP)
      CINTG=CMPLX((RF-RS)*CMPCP,0.0)*CGPOR(XXXXXX)
      IF(ISELF .EQ. 0) GO TO 10
      ANUM=2.0*(TF-TP)*DEL*SINGS
      DEN=(((TF-TP)*DEL)**2+(RF*PHIP)**2)**1.5
      CINTG=CINTG+CMPLX(ANUM/DEN,0.0)
   10 CG6=CINTG
      RETURN
      END
      COMPLEX FUNCTION CANALY(TL,TU,I,ISWR)
C
C----- ROUTINE TO ADD RESULT OF ANALYTICAL INTEGRATION OF THE
C       SINGULAR PORTION OF THE INTEGRANDS
C
      COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SINGS,ACOSGS,GM
      COMMON/FRQ/PI,AK1,AK2,SL1,SL2,OMEGA
      ID=I+1
      TUMTDT=(TU-TF)*DEL
      TLMTDT=(TL-TF)*DEL
      RPI=RF*PI
      IF(TUMTDT) 10, 10, 20
   10 TU1=0.0
      TU2=0.0
      GO TO 30
   20 RAD=SQRT(TUMTDT**2+RPI**2)
      TU1=ALOG(RPI+RAD)
      TU2=ALOG(TUMTDT)
   30 IF(TLMTDT) 50, 40, 40
   40 TL1=0.0
      TL2=0.0
      GO TO 60
   50 RAD=SQRT(TLMTDT**2+RPI**2)
      TL1=ALOG(RAD+RPI)
      TL2=ALOG(-TLMTDT)
   60 TERM=(TU-TF)*(TU1-TU2)+(TF-TL)*(TL1-TL2)
      TERM= -TERM/(PI*RF)
      IF(ISWR .EQ. 0) TERM=TERM/RF
      GO TO (70, 100, 100, 80, 100, 90) ID
```

```
   70 TERM=TERM*2.0*GM/RF
      GO TO 110
   80 TERM=TERM*2.0*GM
      GO TO 110
   90 TERM=TERM/(2.0*RF)
      GO TO 110
  100 TERM=0.0
  110 CANALY=CMPLX(TERM,0.0)
      RETURN
      END


      SUBROUTINE CSMINV(A,NDIM,N,DETERM,COND,IERR)
C***********************************************************************
C
C        CSMINV IS A SUBROUTINE WHICH WILL ACCEPT A SINGLE PRECISION
C        COMPLEX MATRIX AND RETURN THE INVERSE OF THE MATRIX IN ITS
C        PLACE.   THE SUBROUTINE WILL ALSO COMPUTE THE NORMALIZED
C        DETERMINANT OF THE MATRIX, AND THE INVERSE CONDITION NUMBER
C        OF THE MATRIX.
C
C              N     - THE ORDER OF THE MATRIX TO BE INVERTED
C              A     - COMPLEX DOUBLE PRECISION INPUT MATRIX (DESTROYED)
C                        THE INVERSE OF A IS RETURNED IN ITS PLACE.
C              NDIM  - THE SIZE TO WHICH A IS DIMENSIONED IN THE CALLING
C                        PROGRAM
C              DETERM - THE NORMALIZED DETERMINANT OF A WHICH IS RETURNED
C              COND'- THE INVERSE OF MITTRA'S CONDITION NUMBER OF
C                        THE MATRIX.
C              IERR  - ERROR INDICATOR WHOSE VALUE IS ZERO UNLESS TOO
C                        LARGE A MATRIX IS PASSED (.GT. 250 X 250),
C                        IN WHICH CASE IERR=1
C
C        PREPARED BY MICHAEL G. HARRISON E.E. DEPT       JUNE 23, 1972
C
C***********************************************************************
      COMPLEX A(NDIM,NDIM),PIVOT(250),AMAX,T,SWAP,DETERM,U
      INTEGER*4 IPIVOT(250),INDEX(250,2)
      REAL    TEMP,ALPHA(250)
C
C     INITIALIZATION
C
      IERR=0
      IF(NDIM.LE.250) GO TO 5
      IERR=1
      WRITE(5,4) NDIM
    4 FORMAT('0CSMINV ERROR.  ATTEMPT TO INVERT A MATRIX 'I4,
     1' ON A SIDE,'/' WHEN 250 X 250 IS THE MAXIMUM ALLOWED.')
      RETURN
    5 CONTINUE
      DETERM =   CMPLX(1.0,0.0)
      SUMAXA=0.
      DO 20 J=1,N
      ALPHA(J)=0.0
      SUMROW=0.
      DO 10 I=1,N
      ALPHA(J)=ALPHA(J)+A(J,I)* CONJG(A(J,I))
   10 SUMROW=SUMROW + CABS(A(J,I))
      ALPHA(J)= SQRT(ALPHA(J))
```

```
      IF(SUMROW.GT.SUMAXA) SUMAXA=SUMROW
   20 IPIVOT(J)=0
      DO 600 I=1,N
C
C     SEARCH FOR PIVOT ELEMENT
C
      AMAX=CMPLX(0.0,0.0)
      DO 105 J=1,N
      IF (IPIVOT(J)-1) 60, 105, 60
   60 DO 100 K=1,N
      IF (IPIVOT(K)-1) 80, 100, 740
   80 TEMP=AMAX* CONJG(AMAX)-A(J,K)* CONJG(A(J,K))
      IF(TEMP)85,85,100
   85 IROW=J
      ICOLUM=K
      AMAX=A(J,K)
  100 CONTINUE
  105 CONTINUE
      IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1
C
C     INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
C
      IF (IROW-ICOLUM) 140, 260, 140
  140 DETERM=-DETERM
      DO 200 L=1,N
      SWAP=A(IROW,L)
      A(IROW,L)=A(ICOLUM,L)
  200 A(ICOLUM,L)=SWAP
      SWAP=ALPHA(IROW)
      ALPHA(IROW)=ALPHA(ICOLUM)
      ALPHA(ICOLUM)=SWAP
  260 INDEX(I,1)=IROW
      INDEX(I,2)=ICOLUM
      PIVOT(I)=A(ICOLUM,ICOLUM)
      U = PIVOT(I)
      ALPHAI=ALPHA(ICOLUM)
C
C----- THE FOLLOWING SUBROUTINE CALL IS FOR UNDERFLOW PROTECTION DURING
C       CALCULATION OF THE NORMALIZED DETERMINANT
C
      CALL DTRMNT(DETERM,U,ALPHAI)
      TEMP=PIVOT(I)* CONJG(PIVOT(I))
      IF(TEMP)330,720,330
C
C     DIVIDE PIVOT ROW BY PIVOT ELEMENT
C
  330 A(ICOLUM,ICOLUM) =  CMPLX(1.0,0.0)
      DO 350 L=1,N
      U = PIVOT(I)
  350 A(ICOLUM,L) = A(ICOLUM,L)/U
C
C     REDUCE NON-PIVOT ROWS
C
  380 DO 550 L1=1,N
      IF(L1-ICOLUM) 400, 550, 400
  400 T=A(L1,ICOLUM)
      A(L1,ICOLUM)= CMPLX(0.0,0.0)
      DO 450 L=1,N
      U = A(ICOLUM,L)
  450 A(L1,L) = A(L1,L)-U*T
  550 CONTINUE
```

```
  600 CONTINUE
C
C      INTERCHANGE COLUMNS
C
  620 DO 710 I=1,N
      L=N+1-I
      IF (INDEX(L,1)-INDEX(L,2)) 630, 710, 630
  630 JROW=INDEX(L,1)
      JCOLUM=INDEX(L,2)
      DO 705 K=1,N
      SWAP=A(K,JROW)
      A(K,JROW)=A(K,JCOLUM)
      A(K,JCOLUM)=SWAP
  705 CONTINUE
  710 CONTINUE
      SUMAXI=0.
      DO 910 I=1,N
      SUMROW=0.
      DO 900 J=1,N
  900 SUMROW=SUMROW + CABS(A(I,J))
      IF(SUMROW.GT.SUMAXI) SUMAXI=SUMROW
  910 CONTINUE
      COND = 1./(SUMAXA*SUMAXI)
      RETURN
  720 WRITE(5,730)
  730 FORMAT('0',10('*******')/'0MATRIX IS SINGULAR'/'0',10('*******'))
  740 RETURN
      END
      SUBROUTINE DTRMNT(DETERM,U,A)
C***************************************************************************
C
C    THE SOLE PURPOSE OF THIS ROUTINE IS TO SCALE THE NORMALIZED
C    DETERMINANT SO THAT MACHINE UNDERFLOWS WILL NOT OCCUR.
C    THIS IS NECESSARY BECAUSE THE DYNAMIC RANGE OF THE DEC-1077
C    MACHINE IS ONLY ABOUT 10**-38 TO 10**+39.
C
C    THE PARAMETER ISCALE MUST BE RETURNED VIA COMMON TO ANY
C    PROGRAM NEEDING THE VALUE OF THE NORMALIZED DETERMINANT.
C
C    THE VALUE OF THE NORMALIZED DETERMINANT IS THE VALUE RETURNED
C    BY CSMINV TIMES TEN RAISED TO THE POWER (-ISCALE*TEN).
C
C
C    IF CSMINV IS CALLED MORE THAN ONCE BY A PROGRAM, "ISCALE" MUST BE
C    INITIALIZED TO ZERO FOR EACH CALL AFTER THE FIRST.
C
C***************************************************************************
      COMPLEX DETERM,U
      COMMON/SCAFAC/ISCALE
      DATA ISCALE/0/
      IF(CABS(DETERM) .GT. 1.E-10) GO TO 100
      DETERM=DETERM*1.E10
      ISCALE=ISCALE+1
  100 DETERM=DETERM*U/CMPLX(A,0.0)
      RETURN
      END
```

```
      SUBROUTINE CGQ1(CF,XL,XU,N,CVAL)
C***********************************************************************
C
C        PERFORMS INTEGRATION OF A FUNCTION OF A SINGLE VARIABLE
C        BY GAUSSIAN QUADRATURE.
C
C        N = ORDER OF GUASSIAN QUADRATURE APPROXIMATION
C             (2,4,8,10,12,16,32)
C        CF = EXTERNALLY SUPPLIED FUNCTION....MUST BE FUNCTION OF ONE
C              VARIABLE FOR CQG1.
C        XL = LOWER BOUND OF VARIABLE
C        XU = UPPER BOUND OF VARIABLE
C        CVAL = RESULTING VALUE OF THE INTEGRATION
C
C        CF MUST BE LISTED IN AN EXTERNAL STATEMENT
C
C       PREPARED BY MICHAEL G. HARRISON  E.E. DEPT    JUNE 22, 1972
C
C
C***********************************************************************
      IMPLICIT COMPLEX (C)
      DIMENSION Q1(52),Q2(24),Q3(32),NQ(8),NS(8),QG(108)
      EQUIVALENCE (Q1(1),QG(1)),(Q2(1),QG(53)),(Q3(1),QG(77))
      DATA Q1
     $          /.28367513459481288E0,0.5E0,.43056815579702629E0,
     $.17392742256872693E0,.16999052179242813E0,.32607257743127307E0,
     $ 0.48014492824876812E0,.50614268145188130E-1,.39833323870681337E0,
     $.11119051722668724E0,.26276620495816449E0,.15685332293894364E0,
     $.91717321247824900E-1,.18134189168918099E0,.48695326425858586E0,
     $.33335672154344070E-1,.43253168334449225E0,.74725674575290300E-1,
     $.33970478414961227E0,.10954318125799102E0,.21669769706462360E0,
     $.13463335915499818E0,.74437169490815605E-1,.14776211235737644E0,
     $0.49078031712335963E0,.23587668193255914E-1,.45205862818523743E0,
     $.53469662997659215E-1,.38495133709715234E0,.80039164271673111E-1,
     $.29365897714330872E0,.10158371336153296E0,.18391574949909010E0,
     $.11674626826917740E0,.62616704255734458E-1,.12457352290670139E0,
     $.49470046749582497E0..13576229705877047E-1,.47228751153661629E0,
     $.31126761969323946E-1,.43281560119391587E0,.47579255841246392E-1,
     $.37770220417750152E0,.62314485627766936E-1,.30893812220132187E0,
     $.74797994408288370E-1,.22900838882861369E0,.84578259697501270E-1,
     $.14080177538962946E0,.91301707522461790E-1,.47506254918818720E-1,
     $.94725305227534250E-1 /
      DATA Q2
     $             / 0.49759360999851068E+0    ,0.61706148999935998E-2 ,
     *              0.48736427798565475E+0    ,  0.14265694314466832E-1 ,
     *              0.46913727600136638E+0    ,  0.22138719408709903E-1 ,
     *              0.44320776350220052E+0    ,  0.29649292457718890E-1 ,
     *              0.41000099298695146E+0    ,  0.36673240705540153E-1 ,
     *              0.37006209578927718E+0    ,  0.43095080765976638E-1 ,
     *              0.32404682596848778E+0    ,  0.48809326052056944E-1 ,
     *              0.27271073569441977E+0    ,  0.53722135057982817E-1 ,
     *              0.21689675381302257E+0    ,  0.57752834026862801E-1 ,
     *              0.15752133974808169E+0    ,  0.60835236463901696E-1 ,
     *              0.95559433736808150E-1    ,  0.62918728173414148E-1 ,
     *              0.32028446431302813E-1    ,  0.63969097673376078E-1 /
      DATA Q3
     $             /0.49863193092474078E+0 ,  0.35093050047350483E-2 ,
     *              0.49280575577263417E+0    ,  0.81371973654528350E-2 ,
     *              0.48238112779375322E+0    ,  0.12696032654631030E-1 ,
     *              0.46745303796886984E+0    ,  0.17136931456510717E-1 ,
     *              0.44816057788302606E+0    ,  0.21417949011113340E-1 ,
```

```
     *               0.4246838068662849 9E+0    ,    0.2549902963118808 8E-1 ,Q
     *               0 3972418979839712 0E+0    ,    0.2934204673926777 4E-1 ,Q
     *               0.3660910593701448 4E+0    ,    0.3291111138818092 3E-1 ,Q
     *               0.3315221334651076 0E+0    ,    0.3617289705442425 3E-1 ,Q
     *               0.2938578786203811 6E+0    ,    0.3909694789353515 3E-1 ,Q
     *               0.2534499544661147 0E+0    ,    0.4165596211347337 8E-1 ,Q
     *               0.2106756380653176 7E+0    ,    0.4382604650220190 6E-1 ,Q
     *               0.1659343011410638 2E+0    ,    0.4558693934788194 2E-1 ,Q
     *               0.1196436811260685 4E+0    ,    0.4692219954040228 3E-1 ,Q
     *               0.7223598079139825 0E-1    ,    0.4781936003963743 0E-1 ,Q
     *               0.2415383284386915 8E-1    ,    0.4827004425736390 0E-1 /Q
     $,NQ/2,4,8,10,12,16,24,32/,
     $ NS/1,3,7,15,25,37,53,77/
      DO 300 L=1,8
      IF(N.EQ.NQ(L)) GO TO 301
  300 CONTINUE
      WRITE(5,905) N
  905 FORMAT('0 CALLING PARAMETER =',I5,'   INTEGRATION NOT POSSIBLE'//)
      RETURN
  301 CONTINUE
      NP=NS(L)
      NE=NP+N-1
      AX=0.5*(XU+XL)
      BX=XU-XL
      CVAL=(0.,0.)
      DO 350 J=NP,NE,2
      DX=QG(J)*BX
      CVAL=CVAL+QG(J+1)*(CF(AX+DX)+CF(AX-DX))
  350 CONTINUE
      CVAL=CVAL*BX
      RETURN
      END
```

```
      SUBROUTINE CGQ1T(CF,XL,XU,N,CVAL,CVALR)
C**********************************************************************
C
C     SUBROUTINE CGQ1T IS A VERSION OF THE GAUSSIAN QUADRATURE
C     INTEGRATION ROUTINE "CGQ1" WHICH HAS BEEN MODIFIED TO
C     SIMULTANEOUSLY INTEGRATE THE FUNCTIONS (CF) AND ((RHO-PRIME)*CF).
C
C     THE ARGUMENTS ARE THE SAME EXCEPT FOR THE INCLUSION OF THE
C     ADDITIONAL RESULT "CVALR."
C
C**********************************************************************
      IMPLICIT COMPLEX (C)
      DIMENSION Q1(52),Q2(24),Q3(32),NQ(8),NS(8),QG(108)
      REAL COSGS
      COMMON/FNC/RF,ZF,TF,RSL,ZSL,DEL,SINGS,COSGS,GM
      EQUIVALENCE (Q1(1),QG(1)),(Q2(1),QG(53)),(Q3(1),QG(77))
      DATA Q1
     $          /.2886751345948128 82E0,0.5E0,.4305681557970262 9E0,
     $.1739274225687269 3E0,.1699905217924281 3E0,.3260725774312730 7E0,
     $ 0.4801449282487681 2E0,.5061426814518813 0E-1,.3983332387068133 7E0,
     $.1111905172266872 4E0,.2627662049581644 9E0,.1568533229389436 4E0,
     $.9171732124782490E-1,.1813418916891809 9E0,.4869532642585858 6E0,
     $.3333567215434407E-1,.4325316833444922 5E0,.7472567457529030E-1,
     $.3397047841496122E0,.1095431812579910 2E0,.2166976970646236 E0,
     $.1346333591549981 8E0,.7443716949081560 5E-1,.1477621123573764 E0,
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     $0.4907803171233596 3E0,.23587668193255914E-1,.45205862818523743E0,
     $.53469662997659215E-1,.38495133709715234E0,.80039164271673311E-1,
     $.29365897714330872E0,.10158371336153296E0,.18391574949909010E0,
     $.11674626826917740E0,.62616704255734458E-1,.12457352290670139E0,
     $.49470046749582497E0,.13576229705877047E-1,.47228751153661629E0,
     $.31126761969323946E-1,.43281560119391587E0,.47579255841246392E-1,
     $.37770220417750152E0,.62314485627766936E-1,.30893812220132187E0,
     $.74797994408288837E-1,.22900838882861369E0,.84578259697501 27E-1,
     $.14080177538962946E0,.91301707522461 79E-1,.47506254918818720E-1,
     $.94725305227534 25E-1 /
      DATA Q2
     $                / 0.49759360999851068E+0   ,0.61706148999935998E-2 ,
     *               0.40736427798565475E+0    ,  0.14265694314466832E-1 ,
     *               0.46913727600136638E+0    ,  0.22138719408709903E-1 ,
     *               0.44320776350220052E+0    ,  0.29649292457718890E-1 ,
     *               0.41000099298695146E+0    ,  0.36673240705540153E-1 ,
     *               0.37006209578927718E+0    ,  0.43095080765976638E-1 ,
     *               0.32404682596848778E+0    ,  0.48809326052056944E-1 ,
     *               0.27271073569441977E+0    ,  0.53722135057982817E-1 ,
     *               0.21689675381302257E+0    ,  0.57752634026862801E-1 ,
     *               0.15752133984808169E+0    ,  0.60835236463901696E-1 ,
     *               0.95539433736808150E-1    ,  0.62918728173414148E-1 ,
     *               0.32028446431302813E-1    ,  0.63969097673376078E-1 /
      DATA Q3
     $               /0.49863193092474078E+0 ,  0.35093050047350483E-2 ,
     *               0.49280575577263417E+0    ,  0.81371973654528350E-2 ,
     *               0.48238112779375322E+0    ,  0.12696032654631030E-1 ,
     *               0.46745303796886984E+0    ,  0.17136931455510717E-1 ,
     *               0.44816057788302606E+0    ,  0.21417949011113340E-1 ,
     *               0.42468380686628499E+0    ,  0.25499029631188088E-1 ,
     *               0.39724189798397120E+0    ,  0.29342046739267774E-1 ,
     *               0.36609105937014484E+0    ,  0.32911111388180923E-1 ,
     *               0.33152213346510760E+0    ,  0.36172897054424253E-1 ,
     *               0.29385787862038116E+0    ,  0.39096947893535153E-1 ,
     *               0.25344995446611470E+0    ,  0.41655962113473378E-1 ,
     *               0.21067563806531767E+0    ,  0.43826046502201906E-1 ,
     *               0.16593430114106382E+0    ,  0.45586939347881942E-1 ,
     *               0.11964368112606854E+0    ,  0.46922199540402283E-1 ,
     *               0.72235980791398250E-1    ,  0.47819360039637430E-1 ,
     *               0.24153832843869158E-1    ,  0.48270044257363900E-1 /
     $,NQ/2,4,8,10,12,16,24,32/,
     $ NS/1,3,7,15,25,37,53,77/
      DO 300 L=1,8
      IF(N.EQ.NQ(L)) GO TO 301
300   CONTINUE
      WRITE(5,905) N
905   FORMAT('0 CALLING PARAMETER =',I5,'   INTEGRATION NOT POSSIBLE'//)
      RETURN
301   CONTINUE
      NP=NS(L)
      NE=NP+N-1
      AX=0.5*(XU+XL)
      BX=XU-XL
      CVAL=(0.,0.)
      CVALR=CMPLX(0.,0.)
      DO 350 J=NP,NE,2
      DX=QG(J)*BX
      AXP=AX+DX
      AXM=AX-DX
      RSP=RSL+AXP*DEL*SINGS
      RSM=RSL+AXM*DEL*SINGS
```

```
      CFP=CF(AXP)
      CFM=CF(AXM)
      CVAL=CVAL+QG(J+1)*(CFP+CFM)
      CVALR=CVALR+QG(J+1)*(CFP*CMPLX(RSP,0 0)+CFM*CMPLX(RSM,0.0))
  350 CONTINUE
      CVAL=CVAL*BX
      CVALR=CVALR*BX
      RETURN
      END


      SUBROUTINE ICRMUL(CM,CV,CVS,NMROWS,NMCOLS,NVROWS,NVCOLS)
C*************************************************************************
C
C              PERFORMS  [CVS] = [CM] X [CV]
C
C              CM = INPUT MATRIX (NMROWS X NMCOLS)
C              CV = INPUT MATRIX (NVROWS X NVCOLS)
C              CVS = SOLUTION MATRIX (NMROWS X NVCOLS)
C              NMROWS = # OF ROWS IN [CM]
C              NMCOLS = # OF COLUMNS IN [CM]
C              NVROWS = # OF ROWS IN [CV]
C              NVCOLS = # OF COLUMNS IN [CV]
C
C*************************************************************************
      IMPLICIT COMPLEX (C)
      DIMENSION CV(NVROWS,NVCOLS),CM(NMROWS,NMCOLS),CVS(NMROWS,NVCOLS)
      IF(NMCOLS.NE.NVROWS) GO TO 40
      C0= CMPLX(0.0,0.0)
      DO 30 J=1,NVCOLS
      DO 20 I=1,NMROWS
      CSUM=C0
      DO 10 K=1,NMCOLS
   10 CSUM=CSUM+CM(I,K)*CV(K,J)
   20 CVS(I,J)=CSUM
   30 CONTINUE
      RETURN
   40 WRITE(5,10000) NMCOLS,NVROWS
10000 FORMAT(' MULTIPLICATION NOT POSSIBLE.'/
     $' NMCOLS = 'I,10X,'NVROWS = 'I)
      STOP
      END


      SUBROUTINE  BESSEL (X,N,BJ,BY,NX)
      IMPLICIT  REAL*8 (A-H,O-Z)
      DIMENSION BJ(1),BY(1)
      REAL*4 SNGL
      D = 1. D-07
C
C     CHECK FOR ERRORS IN N AND X
C
      IF (N) 710,720,720
710   WRITE (5,900)
900   FORMAT ( 30H   N IS NEGATIVE                )
      GO TO 999
720   IF (X) 730,730,740
```

```
730    WRITE (5,910)
910    FORMAT  (30H   X IS NEGATIVE OR ZERO                    )
       GO TO 999
740    IF (NX .EQ. 1 ) GO TO 30
       IF (NX .EQ. 2) GO TO  600
       IF (NX .EQ. 3)  GO TO 30
       WRITE (5,920)
920    FORMAT  (30H  ERROR IN PARAMETER NX             )
999    WRITE  (5,930)   X,N,NX
930    FORMAT  (30H  PARAMETER VALUES  X,N,NX      ,G15.5,2I5 )
       STOP
C
C CALCULATION OF J BESSEL FUNCTION
C
30     L=N
   31 IF(X-15.)32,32,34
   32 NTEST=20.+10.*X-X** 2. / 3.
       GO TO 36
   34 NTEST=90.+X/2.
   36 IF(L-NTEST)40,38,38
38     WRITE ( 5,940)
940    FORMAT  (30H  RANGE OF X VIOLATED               )
       GO TO 999
40     N1 = L + 1
       BJ(N1)=0.0
       BPREV=.0
C
C      COMPUTE STARTING VALUE OF M
C
       IF(X-5.)50,60,60
   50 MA=X+6.
       GO TO70
   60 MA=1.4*X+60./X
70     MB = L + IFIX ( SNGL (X) )/4 +2
       MZERO=MAX0(MA,MB)
C
C      SET UPE\9L\\\\TOo6MM
C
       MMAX=NTEST
  100 DO 190 M=MZERO,MMAX,3
C
C      SET F(M),F(M-1)
C
       FM1=1.0E-28
       FM=.0
       ALPHA=.0
       IF(M-(M/2)*2)120,110,120
  110 JT=-1
       GO TO 130
  120 JT=1
  130 M2=M-2
       DO 160 K=1,M2
       MK=M-K
       BMK = 2.*DFLOAT(MK)*FM1/X-FM
       FM=FM1
       FM1=BMK
       IF(MK-L-1)150,140,150
  140 BJ(L+1)=BMK
  150 JT=-JT
       S=1+JT
  160 ALPHA=ALPHA+BMK*S
```

```
      BMK=2.*FM1/X-FM
      IF(L)180,170,180
  170 BJ(L+1)=BMK
  180 ALPHA=ALPHA+BMK
      BJ(L+1)=BJ(L+1)/ALPHA
      ERR=BJ(L+1)-BPREV
      IF(DABS(ERR)-DABS(D*BJ(L+1)))200,200,185
  185 BPREV=BJ(L+1)
190   ERR=ERR/BJ(L+1)*100.
      WRITE (5,950) ERR
950   FORMAT(59H  REQUIRED ACCURACY IN BJN NOT OBTAINED, PER CENT DIFF.
     1IS ,G20.6)
200   IF(L-N)210,220,220
220   L=L-1
      IF(L .LT. 0)GO TO 240
      GO TO 31
210   IF(L .EQ. 0)GO TO 240
      DO 230 I=2,N
      L=N-I
230   BJ(L+1)=2.*DFLOAT(L+1)*BJ(L+2)/X-BJ(L+3)
240   IF (NX .EQ. 3) GO TO 600
      RETURN
C
C  CALCULATION OF Y BESSEL FUNCTION
C
600   PI = 3.141592653
C
C     BRANCH IF X LESS THAN OR EQUAL 4
C
      IF (X-4.) 640,640,630
C
C       COMPUTE Y0 AND Y1 FOR X GREATER THAN 4
C
630   T=4./X
      P0=.3989422793
      Q0=-.0124669441
      P1=.3989422819
      Q1=.0374008364
      A=T*T
      B=A
      P0=P0-.0017530620*A
      Q0=Q0+.0004564324*A
      P1=P1+.029218256*A
      Q1=Q1-.00063904*A
      A=A*A
      P0=P0+.00017343*A
      Q0=Q0-.0000869791*A
      P1=P1-.000223203*A
      Q1=Q1+.0001064741*A
      A=A*B
      P0=P0-.0000487613*A
      Q0=Q0+.0000342468*A
      P1=P1+.0000580759*A
      Q1=Q1-.0000398708*A
      A=A*B
      P0=P0+.0000173565*A
      Q0=Q0-.0000142078*A
      P1=P1-.000020092*A
      Q1=Q1+.00001622*A
      A=A*B
      P0=P0-.0000037043*A
```

```
      Q0=Q0+.0000032312*A
      P1=P1+.0000042414*A
      Q1=Q1-.0000036594*A
      A = DSQRT(2.*PI)
      B=4.*A
      P0=A*P0
      Q0=B*Q0/X
      P1=A*P1
      Q1=B*Q1/X
      A=X-PI/4.
      B = DSQRT ( 2./(PI*X))
      Y0 = B*(P0*DSIN(A)+Q0*DCOS(A))
      Y1 = B*(-P1*DCOS(A)+Q1*DSIN(A))
      GO TO 690
C
C       COMPUTE Y0 AND Y1 FOR X LESS THAN OR EQUAL TO 4
C
640   XX=X/2.
      X2=XX*XX
      T=DLOG(XX)+.5772156649
      SUM=0.
      TERM=T
      Y0=T
      DO 670 L = 1,15
      IF(L-1) 650,660,650
650   SUM=SUM+1./DFLOAT(L-1)
660   FL=L
      TS=T-SUM
      TERM=(TERM*(-X2)/FL**2)*(1.-1./(FL*TS))
670   Y0=Y0+TERM
      TERM = XX*(T-.5)
      SUM=0.
      Y1=TERM
      DO 680 L = 2,16
      SUM=SUM+1./DFLOAT(L-1)
      FL=L
      FL1=FL-1.
      TS=T-SUM
      TERM=(TERM*(-X2)/(FL1*FL))*((TS-.5/FL)/(TS+.5/FL1))
680   Y1=Y1+TERM
      PI2=2./PI
      Y0=PI2*Y0
      Y1=-PI2/X+PI2*Y1
C
C     CHECK IF ONLY Y0 OR Y1 IS DESIRED
C
690   IF(N-1) 500,500,530
C
C     RETURN EITHER Y0 OR Y1 AS REQUIRED
C
500   IF(N) 510,520,510
510   BY(2)=Y1
520   BY(1)=Y0
      RETURN
C
C    PERFORM RECURRENCE OPERATIONS TO FIND YN(X)
C
530   BY(1)=Y0
      BY(2)=Y1
      DO 545 K=2,N
      T=DFLOAT(2*(K-1))/X
```

```
      BY(K+1)=T*BY(K)-BY(K-1)
C
C     BESSEL Y FUNCTION HAS EXCEEDED 10**35
C
      IF ( DABS(BY(K+1))-1.0D35) 545,545,541
541   WRITE (5,980)
980   FORMAT  (30H   Y BESSEL FCN EXCEEDED 10**35   )
      GO TO 999
545   CONTINUE
      RETURN
      END


      FUNCTION ELIC1K(AM1)
C***********************************************************************
C
C      COMPLETE ELLIPTIC INTEGRAL OF THE FIRST KIND  K(M),
C       WHERE AM1=1-M
C
C      REFERENCE:
C
C         ABRAMOWITZ AND STEGUN,  EQ.  17.3.34
C
C         MAGNITUDE(ERROR) .LE. 2.0E-8
C
C***********************************************************************
      DATA A0,A1,A2,A3,A4,B0,B1,B2,B3,B4/
     $   1.38629436112,   .09666344259,    .03590092383,  .03742563713,
     $    .01451196212,   .5,              .12498593597,   .06880248576,
     $    .03328355346,   .00441787012/
      A=A0+A1*AM1
      B=B0+B1*AM1
      IF(AM1 .LT. 1.E-18) GO TO 10
      AM12=AM1*AM1
      A=A+A2*AM12
      B=B+B2*AM12
      IF(AM1 .LT. 1.E-12) GO TO 10
      AM13=AM12*AM1
      A=A+A3*AM13
      B=B+B3*AM13
      IF(AM1 .LT. 1.E-9) GO TO 10
      AM14=AM13*AM1
      A=A+A4*AM14
      B=B+B4*AM14
   10 CONTINUE
      ELIC1K=A-B*ALOG(AM1)
      RETURN
      END


      SUBROUTINE TRPADP(CF,XLOW,XHIGH,ER,MAXP,IER,CANS)
C***********************************************************************
C
C      INTEGRATION OF A FUNCTION BY ADAPTIVE TRAPEZOIDAL RULE
C
C
C       CF    = EXTERNALLY SUPPLIED COMPLEX FUNCTION TO BE INTEGRATED
C
C       XLOW  = LOWER LIMIT OF INTEGRATION
```

```
C
C       XHIGH = ((UPPER LIMIT OF INTEGRATION MINUS LOWER LIMIT) DIVIDED
C                BY TWO) + LOWER LIMIT
C
C       ER    = RELATIVE ERROR CRITERION FOR CONVERGENCE CHECK
C
C       MAXP  = MAXIMUM NUMBER OF POINTS TO BE USED
C
C       IER   = ERROR CODE:
C                 = 1, IF INTEGRATION DID NOT CONVERGE WITHIN (MAXP)
C                      POINTS
C                 = 0, OTHERWISE
C
C       CANS  = RESULT OF THE INTEGRATION
C
C
C      CF MUST BE LISTED IN AN EXTERNAL STATEMENT IN THE
C      CALLING PROGRAM.
C
C      THE RESULT (CANS) IS INTEGRAL(CF) BETWEEN THE LIMITS (XLOW) AND
C      (XLOW+(XHIGH-XLOW)*2).  SYMMETRY OF THE FUNCTION (CF) IS ASSUMED
C      ABOUT THE POINT (XHIGH).
C
C*********************************************************************
      IMPLICIT COMPLEX(C)
      COMMON/NPOINT/NP
      IER=0
      DELO=XHIGH-XLOW
      NPOLD=2
      C2=CMPLX(2.0,0.0)
      IF(MAXP .EQ. 0) MAXP=10000
      IF(ER .EQ. 0) ER=1.E-4
      CDELO=CMPLX(DELO,0.0)
      COLD=(CF(XLOW)+CF(XHIGH))/C2
      C=COLD
   10 NP=NPOLD+NPOLD-1
      DEL=DELO/2.0
      CDEL=CMPLX(DEL,0.0)
      K=NP-NPOLD
      K=K+K
      DO 20 J=2,K,2
      X=XLOW+DEL*FLOAT(J-1)
   20 C=C+CF(X)
      CO=COLD*CDELO
      CN=C*CDEL
      CND=CN
      ABSND=CABS(CND)
      IF(ABSND .LT. 1.E-20) ABSND=1.E-20
      DIFF=CABS(CN-CO)
      ERR=DIFF/ABSND
      IF(ERR .LT. ER) GO TO 40
      IF(NP .GE. MAXP) GO TO 30
      DELO=DEL
      CDELO=CDEL
      NPOLD=NP
      COLD=C
      GO TO 10
   30 IER=1
   40 CANS=CN*C2
      RETURN
      END
```

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